

The impact of deunionization on the growth and dispersion of productivity and pay

Giovanni Dosi^{1*}, Richard B. Freeman², Marcelo C. Pereira^{1,3},
Andrea Roventini¹, Maria Enrica Virgillito¹

¹Institute of Economics and EMbeDS, Scuola Superiore Sant'Anna, Piazza Martiri della Libertà 33, I-56127, Pisa, Italy. e-mail: gdosi@santannapisa.it, m.virgillito@santannapisa.it, ²Harvard University and National Bureau of Economic Research, 1050 Massachusetts Avenue, Cambridge, MA, USA. e-mail: freeman@nber.org, ³Institute of Economics, University of Campinas, Rua Pitágoras 353, 13083-970, Campinas, SP, Brazil. e-mail: mcper@unicamp.br and ⁴OFCE, Sciences Po, Nice France. e-mail: andrea.roventini@santannapisa.it

*Main author for correspondence.

Abstract

This article presents an Agent-Based Model (ABM) that seeks to explain the concordance of sluggish growth of productivity and of real wages found in macroeconomic statistics, and the increased dispersion of firm productivity and worker earnings found in micro level statistics in advanced economies at the turn of the 21st century. It shows that a single market process unleashed by the decline of unionization can account for both the macro- and micro-economic phenomena, and that deunionization can be modeled as an endogenous outcome of competition between high wage firms seeking to raise productive capacity and low productivity firms seeking to cut wages. The model highlights the antipodal competitive dynamics between a “winner-takes-all economy” in which corporate strategies focused on cost reductions lead to divergence in productivity and wages and a “social market economy” in which competition rewards the accumulation of firm-level capabilities and worker skills with a more egalitarian wage structure.

JEL classification: J51, E02, E24, C63

1. Introduction

In the first two decades of the 21st century, advanced capitalist economies experienced a striking set of changes in macroeconomic, microeconomic, and institutional patterns that merit explanation. At the macro level, productivity growth slackened and real wage growth decoupled from the productivity growth to increase at far lower rates.¹ At

- 1 On the aggregate facts, [Syverson \(2017\)](#) shows that US labor productivity growth fell from 2.8% in 1995–2004 to 1.3% in 2005–2015 and that 29 out of the 30 advanced countries had similar declines; [Hutchinson and Persyn \(2012\)](#) and [Karabarbounis and Neiman \(2014\)](#) document the falling labor share of income in developed countries; [Schwellnus et al. \(2017\)](#) show that the fall was accompanied by an increasing ratio of the mean wage to the median wage income that reflects a widening distribution of wage income. On the micro facts, [Dunne et al. \(2004\)](#) estimate a sizeable firm

the micro level, inequality increased among firms in productivity and in the average earnings paid to workers. Over the same period, the institutional structure of labor markets changed, with the proportion of workers covered by unions shrinking nearly everywhere.² Empirical studies document these patterns in enough data sets and countries to establish them as *stylized facts* for theory and models of how capitalism operates in a modern economy.

What explains the concordance of sluggish macroeconomic performance and divergence in microeconomic firm outcomes? What connection, if any, exists between those patterns and declining unionization?³

This article presents an Agent Based Model (ABM) of the dynamics of productivity growth and wage determination in labor markets with and without unions that offers a unified explanation for all above patterns. The model simulates an economic world in which firms endowed with heterogeneous productive capacity compete under two opposite labor relation systems: one that we label as “non-union” in which firms unilaterally set wages and hire and fire workers and one that we label as “union” in which firms pay the same collectively determined wage for similar workers, and follow a collective agreement in hiring and firing. The non-union system pressures low-productivity firms to cut wages to survive. The union system pressures firms to invest in productivity-enhancing activities to survive. The differential pressure makes the mode of wage setting a selection mechanism among firms with heterogeneous strategies of squeezing worker pay vs enhancing worker and firm competence. The model accounts for the deunionization, sluggish growth of productivity and real wage and increased dispersion of firm pay and productivity of advanced countries in terms of micro market behaviors rather than as independent phenomena following their own dynamics.

ABMs are well-suited to analyze the concordance of micro, macro, and institutional developments. They differ from stochastic general equilibrium models by being open-ended simulations driven by heterogeneous agents who follow simple bounded rationality rules of behavior in disequilibrium situations, rather than being closed-form solutions derived from linearization around equilibrium conditions. The rules govern the *internal* growth or decline of firms over time and the *entry* and *exit* of firms that together change micro productivity and pay, and aggregates to macro and institutional outcomes.

A recurrent concern with ABMs is that analysts who focus on explaining a few stylized facts may “overfit” the model with many behavioral rules designed for those facts but which would not hold generally. Building our analysis on the “Schumpeter meeting Keynes” (K + S) family of models that robustly accounts for a large ensemble of stylized facts at micro- and macro-levels (see references to Table 2), we avoid this problem. We further conduct robustness tests of the simulations to a range of different values of parameters.

2. Dispersion of productivity and wages and the dynamics of deunionization

Our agent-based modeling situates micro behavior in market settings where competition imposes evolutionary pressure on agents. The main agents are firms whose characteristics and behavior give them differing market rewards. Those with higher rewards prosper and expand in the market while those with lower rewards see their share of market outcomes shrink. The firms are heterogeneous in their productivity but have limited ways to learn and adjust behavior.

The assumption of firm heterogeneity is predicated on evidence that documents the overweening importance of heterogeneity in productivity and wages among firms and the establishments where they conduct business. The data show:

- Wide dispersion among firms in productivity and average pay in a given period (Dunne *et al.*, 2004) and in changes in productivity and pay even among demographically identical workers in narrowly defined industries and occupations (Barth *et al.*, 2016).

contribution to wage dispersion, Barth *et al.* (2016) show that increased inter-establishment wage dispersion contributes roughly twice as much to the growth of wage dispersion in the US as the increase in intra-establishment inequality. Berlingieri *et al.* (2017) document rising dispersion in wages and productivity across 16 OECD countries from the mid-1990s to 2012. While the estimated magnitudes of these effects vary, we know of no empirical evidence that contravenes the stylized patterns.

2 Farber *et al.* (2018) for US, Ebbinghaus and Visser (1999) for EU.

3 See Stansbury and Summers (2020) for supportive empirical evidence.

- Within firm growth of productivity having a bigger impact than reallocation of workers from low- to high-productivity firms on aggregate productivity growth (Dosi *et al.*, 2015).⁴
- Lower dispersion of productivity in economies with compressed wage structures, as the compressed wage structure pushes low productivity companies out of the market (Barth *et al.*, 2014).⁵
- Lower dispersion of pay within and among unionized compared to non-union establishments for workers with similar measured skills and for the same workers who change employment over time (Freeman, 1984).
- An increasing share of “zombie firms” in the US from 2003 to 2013—firms unable to pay outstanding interest but failing to exit the market (McGowan *et al.*, 2017). This failure in market “cleansing” contributes to sluggish aggregate productivity growth.

Taken together the failure of market forces to compress the widening distribution of wages towards central levels even in the US, where union/institutional constraints on market forces are weakest, suggests the need for a model of labor market adjustment that goes beyond the assumption that the institution-less non-union market determines a single market-clearing wage in the long run, if not in the short or intermediate run.

Our ABM model links the macro and micro facts to institutional developments by simulating the interaction of union and non-union wage-setting systems with heterogeneity in the distribution of productivity among firms. Figure 1 shows the essence of the model.⁶ The x axis in each panel ranks firms from the most to the least efficient while the y axis shows how efficiency translates into costs and thus the likely survivability of firms. Figure 1(A) represents an industry with a union-bargained wage that applies to all firms so that the ranking of firms from the most productive (firm 1) to the least productive (firm n) ranks them inversely by unit costs. With all firms paying the same wage to equally skilled workers, the more productive firms have lower costs. If the firms compete in a market with a single price, the dynamics favors them and they expand while less efficient firms shrink. The single bargained wage prevents firms from squeezing wages and thus pressures them to compete on productivity and thus for selection on productivity. In Figure 1(A), union firms 1 through $n - 2$ remain in the market while union firms $n - 1$ and n are driven out, which truncates the distribution of productivity.

Figure 1(B) depicts an industry in the opposite “non-union” situation where firms pay workers a wage indexed on firm-specific productivity. In this case, all firms have the same unit costs of production and survive irrespective of their efficiency. Competition does not favor firms 1, 2, 3 with higher relative productivity as their wages are commensurately higher relative to their productivity. If productivity falls for any reason in a firm, the firm reduces pay to survive, which allows the inefficient firms $n - 2$ to n to stay in business. Dispersion of productivity is bound only by low productivity workers with reservation wages above the wage that would justify their low productivity.

Figure 1(C) analyses competition between unionized firms that pay a single wage dependent on the average productivity of the unionized group and non-unionized firms whose wages depend only on their firm’s productivity. As non-union firms enter, their production reduces the price of the good and pressures the least efficient union firms. In this case, competition can exert a *negative* impact upon the dynamics of mean productivity as union firms with below union-group productivity but higher productivity than non-union firms go out of business while non-union firms with low productivity and low wages survive.

Noting the tendency for labor unions to grow rapidly and decline slowly, Freeman (1997) proposed a spurt model of unionization, which entails a phase transition triggered after some tipping point that leads many workers and firms to unionize in a sudden sharp increase. This is followed by competitive exit and entry forces that can favor non-union firms and gradual reduce union density.⁷ Interpreting unionization as stemming from workers and firms, Bryson *et al.* (2017) argue that deunionization is driven largely by new cohorts of workers/firms who do not unionize rather than by previously unionized workers/firms abandoning the institution. New firms experience a “never-member”

4 The contribution of reallocation seems to further weaken post the Great Recession (Foster *et al.*, 2016), with the rate of entry of new firms falling while exit rates holding steady (Decker *et al.*, 2016).

5 Earlier work by Hibbs and Locking (2000) shows that lower within-plant wage standard deviation is associated with higher productivity levels, with a shift of labor from low- to high-productivity firms.

6 This representation draws on Salter (1960) analysis of the productivities of different vintages of equipment.

7 In this analysis, governments that enact laws favorable to unions, such as the Wagner Act in US, the Blum government in France, or the PC 1003 in Canada are responding to worker pressures in the spurt. The laws are not an exogenous determinant of unionism from the top but a pathway created to “tame” worker unrest.

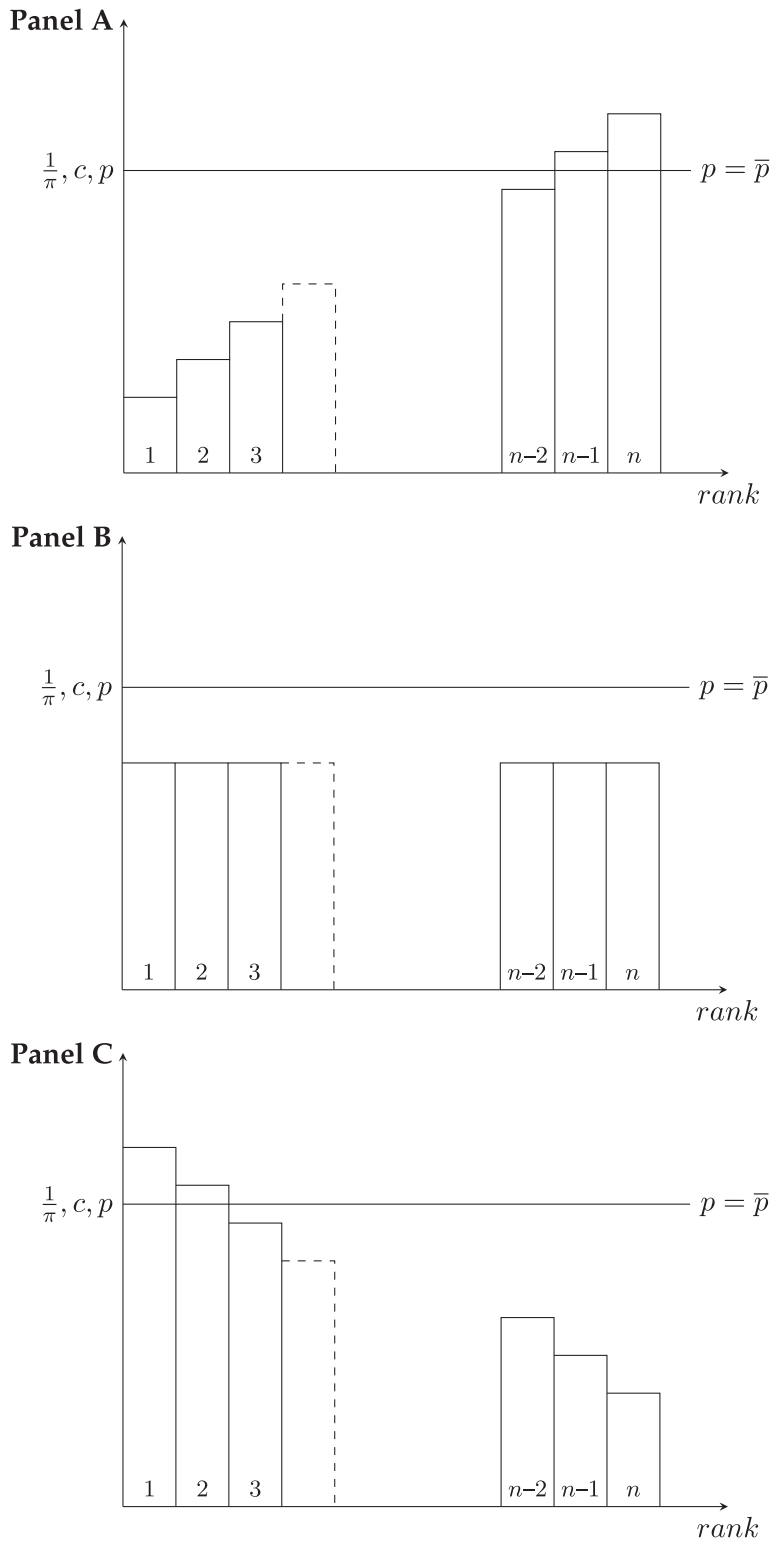


Figure 1. (A) Uniform wage; (B) wages proportional to productivity; (C) different wage elasticities to productivity.

effect as workers with no union experience are unlikely to demand this good/service against a management that prefers to operate non-union. In fact, young workers express greater desire for unions than older workers (Bryson *et al.*, 2005) but are less likely to be able to accomplish that desire within the organization and thus less willing to try to overcome management's strategy to remain non-union.⁸

We begin our simulation model with a completely unionized labor market. Then non-union firms enter. Unionized and non-unionized firms compete in the product market in terms of relative prices, product quality, and quantity (excess or unfilled demand), which determine the average dispersion of productivity and wages. If the share of employment/output in non-union companies increases, the mean level of productivity growth declines while the standard deviation of productivity among firms increases. Competition shifts from improving productivity to reducing wages and cost. Firms whose comparative advantage lies in cutting pay gain at the expense of those whose advantage lies in high productivity. Cost competition induces a deflationary spiral and reduces product quality.

But a simple strategy of reducing wages may not succeed in driving out higher wage/more productive firms. To the extent that lower wages reduce the quality of production or rate of improvement of productivity, the new low wage competitors may fail to survive over the long run even if they increase their share of production in the short run. We use the model to explore the nexus between deunionization and the stylized productivity facts and the factors that determine whether a given labor market will shift its wage-setting system.

3. The model

Figure 2 gives the bare-bones structure of the model. It extends Dosi *et al.* (2017, 2018) variant of the basic K + S artificial economy (Dosi *et al.*, 2010) that included endogenous worker skill accumulation and variable number of firms in a general disequilibrium, stock-and-flow consistent, agent-based model, populated by heterogeneous workers/consumers, capital-good firms, consumption-good firms, and banks, plus the central bank and government.⁹ Agent behavior follows bounded-rational rules. To apply the model to our problem, we add two differentiated mechanisms of job hiring and firing, search process, and wage setting to characterize the type of firm.¹⁰

The model highlights the importance of increasing knowledge in the growth of productivity by dividing firms between those that produce capital-goods and those that produce consumption goods. The capital good firms invest in R&D and produce heterogeneous goods/services/knowledge that raise the productivity of the consumption-good firms. This is the locus of endogenous innovation, characterized by imperfect information and Schumpeterian competition driven by technological change. Given the increased proportion of investment in software and in information, communication, and technology equipment, we view this sector as extending beyond traditional producers of machine tools/equipment to include those developing new software and information, technology, and communication goods and services.

Since creating knowledge depends critically on human activity, we make labor the only factor of production in the capital-goods producing sector. These firms report the price and productivity of their machines and services to current customers and a subset of potential new ones, and invest a fraction of past revenues in R&D aimed at improving their products. They set prices at a fixed mark-up over labor costs. In a typical model run, capital-good firm workers represent less than 10% of the employed labor force, so we focus on labor patterns in the consumption-good sector.

Consumption-good firms combine vintages of capital bought from capital-goods firms with labor to produce a single, quality-differentiated good for consumers under constant returns to scale. Desired production is determined by adaptive demand expectations. Given inventories, if the current capital stock cannot produce the desired output, firms order new machines to expand capacity, funded by retained past profits or, up to a limit, bank loans. They replace old machines according to a payback-period rule. Firms choose the capital-good supplier based on the price and productivity of machines. As new machines embed state-of-the-art technologies, the productivity of consumption-good firms increases over time. Consumption-good firms set prices by a variable mark-up on labor production

8 For evidence that this may be changing among “millennials”, see <https://www.rewire.org/work/younger-workers-labor-unions/>.

9 Subscript t stands for (discrete) time $t = 1, 2, \dots, T$. Agent-specific variables are denoted by subscript ℓ , in case of workers, i , for capital-good firms, j , for consumption-good firms, and k , for banks.

10 The code and a user-friendly interface are accessible at <https://github.com/SantAnnaKS/LSD>.

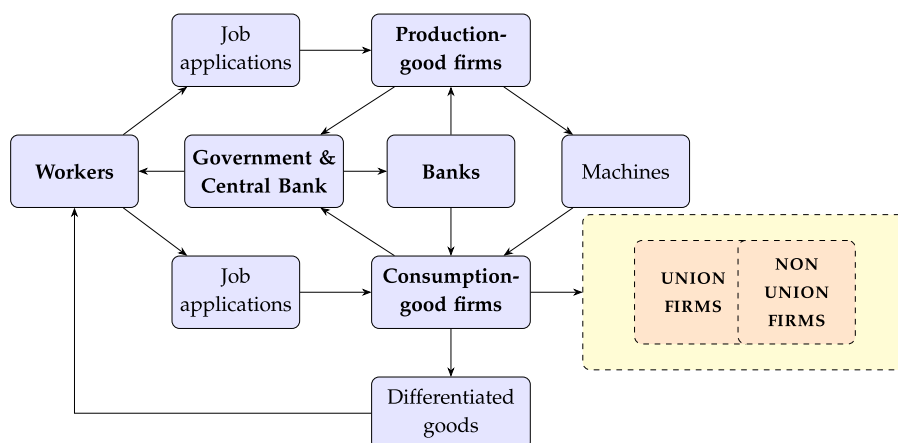


Figure 2. The model overall structure. Boxes in bold style represent the model's agents. Dashed boxes represent the two variants of firms (orange) competing in the product market (yellow).

costs to balance profit margins and market shares, raising (lowering) mark-ups and prices when market shares expand (decline). Due to imperfect information their consumers switch gradually to the most competitive producer so that market shares evolve according to a (quasi) replicator dynamics as more competitive firms expand, while less competitive firms shrink or close down.

Exit and entry of firms is endogenous in both sectors. Firms exit when market shares get close to zero or go bankrupt when net assets turn negative. Conversely, firms enter the market through a stochastic process that depends on the number of incumbents and financial conditions. Entry is easier when the sectoral liquidity-to-debt ratio is high. Banks take deposits and provide interest-paying loans to finance firms' production and investment plans. They allocate credit to firms seeking credit according to a loan to value ratio rule. The supply of credit is elastic.

The labor market is modeled as a decentralized search-and-hire process between workers and firms. Workers search for jobs at a random subset of employers. The unemployed submit job applications to firms. A proportion of employed workers apply for better positions. Larger firms have a proportionally higher probability of receiving job applications, which are organized in firm-specific application queues. Capital-good firms hire workers according to their demands. Consumption-good firms hire workers depending on adaptive demand expectations; while for simplicity, banks, the central bank and the government have no workers. The aggregate supply of labor is fixed and available to be hired in any period.

The labor market is also characterized by imperfect highly localized information. Firms observe workers' skills and wage requests on their own queues, while workers are aware only of the wage offers from firms where they applied for a job. Firms decide whether to hire, fire, or keep the current labor force. Each hiring firm makes a unique wage offer to job applicants, based on economy-wide productivity in the case of union firms and on the received applications in non-union ones. Workers select the best wage offer from firms to which they submitted applications, with employed workers quitting the current job if they receive a better offer.

We treat one round of interactions between workers and firms per period. The overall demand for labor depends on the aggregate demand of the economy, which creates the possibility that the labor market does not clear even absent firing or hiring transaction costs. Firms may fail to fill all open positions, and workers may not find a job even when there are still unfilled positions. Systematic discrepancies between vacancies and involuntary unemployed are likely to be the rule rather than the exception in the aggregate.

Workers spend their income on the consumption good.¹¹ If the supply of the consumer good falls short of demand, excess demand is saved in banks for future consumption. The central bank sets the reserves from the banks and bails out failing banks. The government taxes firm and bank profits, pays unemployment benefits, imposes a

11 The macroeconomic results hold as long as the propensity to consume out of wages is higher than out of profits. Volatility of spending is lowest for consumption, then for GDP, and highest for investment.

minimum wage, absorbs profits and losses from the central bank, and keeps a non-explosive public debt trajectory in the long run.

Two other distinctive features of our model deserve attention. First, firms decide how much to produce extrapolating on their past sales (Dosi *et al.*, 2020). Past equipment is bygone and remains part of firms' resources if the firm does not scrap it. Conversely expansionary investment depends on the fixed coefficient associated with a new machine (and software) offered by the machine-producing sector based on a pay-back rule, which has little to do with elasticity of substitution. This treatment simplified the demand for labor from the notion of choice along a production function.

Second, we do not model "strategic" game theoretic interactions among firms. Instead, we treat organizational traits of behavior as sticky, irrespective of market signals (Dosi *et al.*, 2001). That is Toyota does not become Foxcom under any circumstance and vice versa. In our model, this is reflected by the absence of any firm-level switching rule driven by relative performance. Instead, given their organizational types, firms are selected by competitive forces.

Appendix A contains the details of the model.

3.1 Competition between unionized and non-unionized firms

Table 1 contrasts the wage-setting and other features of union firms and non-union firms. *Unionized firms* pay the same wages to all workers with the same skills and change wages as aggregate and market productivity change. They fire employees only when profits become negative. Their workers seek alternative jobs less frequently than non-union workers, consistent with the exit-voice trade-off in the labor market (Freeman, 1980). In hiring and firing, firms try to keep the more skilled workers. Conversely, *non-unionized* firms set wages according to worker skills and labor market conditions. Wages are set by an asymmetric negotiation process where firms have the last say over workers. There are no hiring/firing protections and unemployed workers adjust downward their "satisfying" wages.

Employed workers search for better paid jobs and firms fire excess workforce according to planned production. Hiring and firing of workers is based on the skills to (individual) wage ratio or just the latter, according to the scenario. The market share of unionized firms is $f_t^u \in [0, 1]$ while that of non-unionized firms is $f_t^n \in [0, 1]$.

To focus on the decline in unionization, we assume that non-union firms enter and compete with union incumbent firms in an evolutionary process. From $t = 100$, the probability of an entrant being non-union is fixed at 50%. The time window ($100 \leq t < 200$) allows non-union entrants to grow and achieve some joint market share. At the end of this period, the likelihood of union or non-union firms entering the consumer-good market is proportional to their relative populations f_{t-1}^u and f_{t-1}^n .

In the consumer-goods sector, firms compete according to their relative cost competitiveness. Firm j market share evolves following a replicator dynamics:

Table 1. Characteristics of the two types of firms

Firms behavior	Union	Non-union
Differentiated wages	No	Yes
Wage sensitivity to unemployment	Low (rigid)	High (flexible)
Wage indexation to average productivity	Full	Partial
Labor-firing restrictions	Under losses only	None
Worker-hiring rule	Higher skills	Depends on scenario
Worker-firing rule	Lower skills	Depends on scenario
Worker new-job search intensity	Low ($\omega = 2$)	High ($\omega = 5$)
Scenarios		
Invasion	Hire (fire) workers with lower (higher) skill first	Hire (fire) workers with lower (higher) wage-to-skill ratio first
No invasion	Hire (fire) workers with lower (higher) skill first	Hire (fire) workers with lower (higher) wage first

$$f_{j,t} = f_{j,t-1} \left(1 + \chi \frac{E_{j,t} - \bar{E}_t}{\bar{E}_t}\right), \quad \bar{E}_t = \frac{1}{F_t^2} \sum_j E_{j,t} f_{j,t-1}, \quad (1)$$

where $\chi \in \mathbb{R}_+$ is a parameter, F_t^2 the current number of firms in the consumer-good market, $\bar{E}_t$ the average competitiveness, and the firm relative competitiveness $E_{j,t}$ is defined by the individual normalized price $p'_{j,t}$, unfilled demand $l'_{j,t}$, and product quality $q'_{j,t}$, with parameters $(\omega_1, \omega_2, \omega_3) \in \mathbb{R}_+^3$:

$$E_{j,t} = -\omega_1 p'_{j,t-1} - \omega_2 l'_{j,t-1} - \omega_3 q'_{j,t-1}. \quad (2)$$

Firms set consumption-good prices by a variable mark-up $\mu_{j,t}$ on average unit cost $c_{j,t}$:

$$p_{j,t} = (1 + \mu_{j,t}) c_{j,t}. \quad (3)$$

Firms' mark-up rule is driven by the evolution of individual market shares with parameter $v \in \mathbb{R}_+$:

$$\mu_{j,t} = \mu_{j,t-1} \left(1 + v \frac{f_{j,t-1} - f_{j,t-2}}{f_{j,t-2}}\right). \quad (4)$$

Unfilled demand $l_{j,t}$ is the difference between actual demand $D_{j,t}$ firm j gets and its effective production $Q_{j,t}$ plus existing inventories $N_{j,t}$ from past periods, if any:

$$l_{j,t} = \max[D_{j,t} - (Q_{j,t} + N_{j,t}), 0]. \quad (5)$$

The quality of consumer-goods produced by firm j is determined by the average (log) skill level of its workers. This captures the notion that firm-specific accumulated skills are more complementary to incremental product innovation.

$$q_{j,t} = \frac{1}{L_{j,t-1}} \sum_{\ell \in \{L_{j,t-1}\}} \log [s_{\ell,t-1}]. \quad (6)$$

The skill of employed workers improves over time while unemployed workers lose skills:

$$s_{\ell,t} = \begin{cases} (1 + \tau_T) s_{\ell,t-1} & \text{if employed in } t-1 \\ \frac{1}{1 + \tau_U} s_{\ell,t-1} & \text{if unemployed in } t-1, \end{cases} \quad (7)$$

where $(\tau_T, \tau_U) \in \mathbb{R}_+^2$ are parameters governing the learning rate while the worker is employed or unemployed. A newly hired worker immediately acquires the minimum skill level present in the firm—the incumbent worker with the lowest skills—if above her present level. Workers have a fixed working life, retiring at a specified point, at which they are replaced in the labor market by young workers with skills at the current minimum level among employed workers. At the beginning of each simulation, initial working ages are randomly drawn in the range $1, 2, \dots, T_r$ (T_r is a parameter) and start from the same skill level.

Worker ℓ current skills $s_{\ell,t}$ define her individual (potential) productivity:

$$A_{\ell,t} = \frac{s_{\ell,t}}{\bar{s}_t} A_i^\tau, \quad (8)$$

being $\bar{s}_t$ the average overall skill level of the economy, and A_i^τ the standard productivity of the specific machinery vintage the worker operates. Thus, the worker's normalized skill represents her productivity relative to that expected for the machine vintage. This makes firm-level effective productivity an emergent property resulting from the supplier-driven introduction of new vintages, evolution of workers' skills, and the demand decisions which guide capital accumulation and vintage mix of machines:

$$A_{j,t} = \frac{1}{L_{j,t-1}} \sum_{\ell \in \{L_{j,t-1}\}} A_{\ell,t}, \quad (9)$$

where $L_{j,t}$ is the number of workers at firm j , and $\{L_{j,t}\}$, the size of this set. So, if the mean wage paid by firm j is $w_{j,t}$, its average unit cost is given by:

$$c_{j,t} = \frac{w_{j,t}}{A_{j,t}}. \quad (10)$$

Finally, we allow for other pay setting institutions through a profit-sharing mechanism which allows firms with above-average profits distribute bonuses. For simplicity, bonuses are equal for all workers in the firm. Thus, the total bonuses by firm are:

$$B_{j,t} = \psi_6(1 - tr)\Pi_{j,t-1}, \quad (11)$$

being $\psi_6 \in [0, 1]$ a sharing parameter, $tr \in [0, 1]$ the tax rate parameter, and $\Pi_{j,t}$ the firm gross profit. Therefore, the total income of worker ℓ working for firm j in period t is $w_{\ell,t} + B_{j,t}/L_{j,t}$.

Appendix A describes the remaining behavioral rules characterizing agents. Appendix C gives model's parameters, initial conditions, and stock-flow matrix. In each simulation period, the following events take place:

INITIATION OF CHANGES

1. Workers (employed and unemployed) update their skills;
2. Machines ordered in the previous period (if any) are delivered;
3. Capital-good firms perform R&D and signal their machines to consumption-good firms;
4. Consumption-good firms determine their desired production, investment and workforce;

RESPONSES TO CHANGES

5. Firms allocate cash-flows and (if needed) borrow from banks to operate and invest;
6. Firms send/receive machine-tool orders for the next period (if applicable);
7. Job-seeking workers send job applications to firms;
8. Wages are set (collective indexation or individual negotiation) and job vacancies are partly or totally filled;
9. Firms pay wages/bonuses and government pays unemployment benefits;

MARKET OUTCOMES

10. Consumption-good market opens and market shares are allocated according to the relative competitiveness of firms;
11. Firms and banks compute their profits, pay taxes, and repay (part of) their debt;
12. Exit takes place, near-zero share, and bankrupt firms leave the market;
13. Prospective entrants decide to enter according to market conditions;
14. Aggregate variables are computed and the cycle restarts.

4. Robustness and interpretative power

As noted, our model builds on earlier variants of the K + S model that generate endogenous growth and business cycles, and fit stylized facts beyond those on which we focus. The top panel of Table 2 lists the stylized facts that the model fits at both the macro- and the micro-economic levels while the bottom panel shows the stylized facts fit by the labor-enhanced version of the model, which explicitly accounts for decentralized firm–worker interactions.

Our model adds union and non-union wage setting and competition as described in Table 1. The simulations produce two key scenarios: (i) successful non-union firms invasion of the previously all union market; and (ii) the setting where the union firms overcome the challenge of new competitors maintaining market dominance.

The difference between the set-ups depends critically on the worker hiring and firing rules of entering non-union firms. In the first scenario, non-union firms consider both worker wage and skills when hiring or firing, which enables them to gain advantage over union firms. In the second scenario, non-union firms just evaluate wages when hiring/firing and fail to overturn the market by being too “lean and mean.” The evidence of deunionization makes the first scenario the realistic one. Indeed, while we have not modeled a union spurt, being too lean and mean could potentially lead to such an event.

The two scenarios yield similar qualitative results for outcomes that were not “built into” the model and that fit stylized facts for labor markets and versions of K + S models which did not build in unionism.¹²

12 All figures (except Figure 8) and tables below are from 100 Monte Carlo (MC) runs of the model. MC runs are required because of the stochastic components in the model. One hundred runs yield narrow confidence intervals for the mean results.

Table 2. Stylized facts matched by the K + S model at different aggregation levels

Microeconomic stylized facts	Aggregate-level stylized facts
Skewed firm size distribution	Endogenous self-sustained growth with persistent fluctuations
Fat-tailed firm growth rates distribution	Fat-tailed GDP growth rate distribution
Heterogeneous productivity across firms	Endogenous volatility of GDP, consumption and investment
Persistent productivity differentials	Cross-correlation of macro variables
Lumpy investment rates of firms	Pro-cyclical aggregate R&D investment and net entry of firms in the market
Heterogeneous skills distribution	Persistent and counter-cyclical unemployment
Fat-tailed unemployment time distribution	Endogenous volatility of productivity, unemployment, vacancy, separation and hiring rates
Fat-tailed wage growth rates distribution	Unemployment and inequality correlation
	Pro-cyclical workers skills accumulation
	Beveridge curve
	Okun curve
	Wage curve
	Matching function

Source: Dosi *et al.* (2017).

Figure 3(a) shows that in both scenarios the firm size-rank distributions in the consumer-good sector (where we apply the analysis) is right skewed with a heavier tail than a fitted lognormal distribution. Figure 3(b) shows a dynamics reasonably consistent with a Gibrat multiplicative process where growth is independent from initial conditions,¹³ while Figure 3(c) shows a Laplace process which relaxes the strong form of the Gibrat's law with i.i.d. growth rates, fitting both scenarios better. While the parameters on the higher moments differ between the two, both robustly display heavy-tailed properties. Figure 3(d) depicts the scaling of (log) standard deviation of the growth rate with respect to firm size. Finally, we also find that productivity is positively autocorrelated in time.¹⁴ All these results are in line with the empirical evidence.¹⁵

The value added of differentiating non-union and union firms is that it gives us a way to analyze the conditions under which non-union firms come to dominate the market with firm-level pay-setting and the impact of that dominance on dispersion of productivity and wages and on productivity growth. In the deunionization scenario, non-union firms consider both worker wage and skills when hiring or firing, which enables them to gain advantage over union firms. In the scenario where unions survive, non-union firms evaluate wages but do not adjust for worker skills when hiring/firing, failing to dominate the union firms by being too “lean and mean.”

Figure 4 shows the “organizational ecologies” in the two scenarios. Figure 4(a) gives the outcome in which the non-union firms dominate, Figure 4(b) shows cases in which the invasion fails due to the (stochastic) competition process between the two types of firms, and the path-dependence in the model. The light grey area represents the maximum and the minimum realizations of the model while the dark grey gives the 95% confidence interval. What explains the differences between the two scenarios? By changing their hiring/firing strategy to hiring lower wage workers and firing high wage workers without taking account of heterogeneity in worker productivity the non-union firms fail to take over the market.

- 13 Figure 3(b) and (d) reports results for the no-invasion scenario however the patterns are qualitatively similar for both.
- 14 AR parameters are approximately equal to 0.9 in both scenarios.
- 15 For discussions on the stylized facts on the dynamics of industries, see Dosi *et al.* (2017), Bottazzi and Secchi (2006), and Calvino *et al.* (2018).

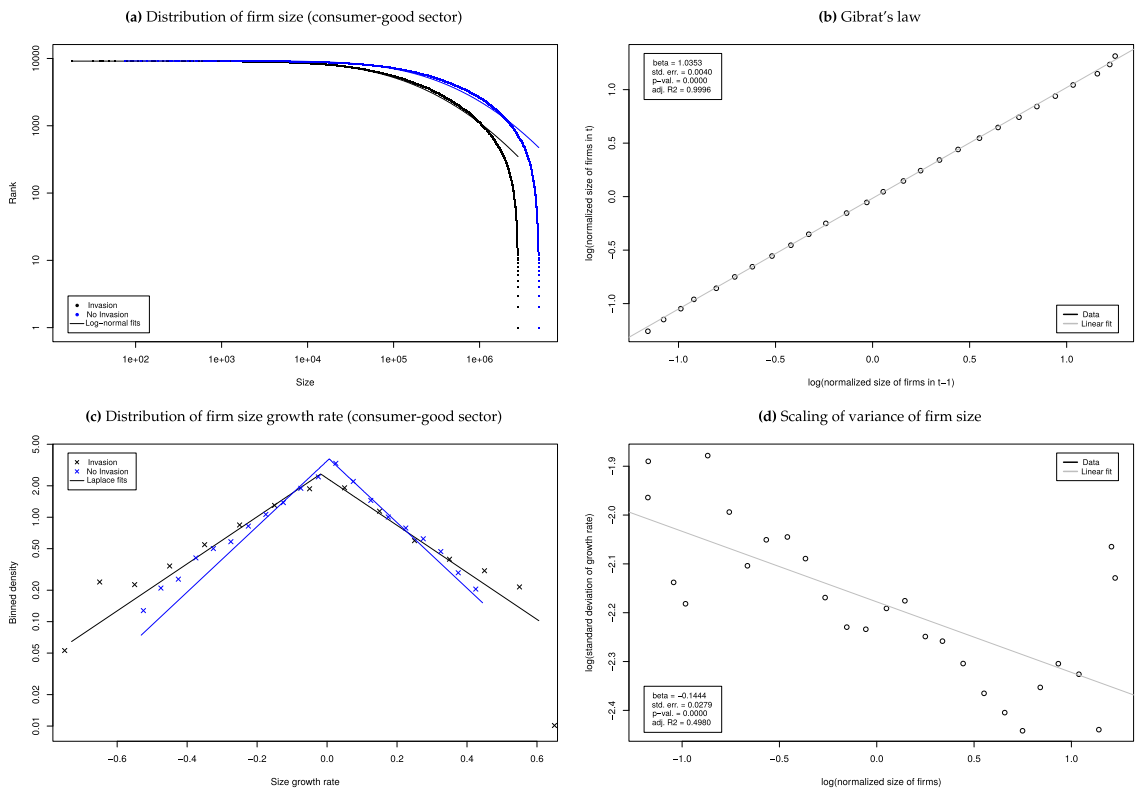


Figure 3. Matching of stylized facts in industrial dynamics. (a) Distribution of firm size (consumer-good sector), (b) Gibrat's law, (c) distribution of firm size growth rate (consumer-good sector), and (d) scaling of variance of firm size. Pooled data from 100 MC runs in period [200, 400].

What happens to wages? Figure 5(a) presents the distribution of firm average wages. It is far less dispersed in the scenario where the majority of firms remain union. This is also true for the distributional width of wage growth rate in Figure 5(b). When union firms prevail, a much more egalitarian wage dynamics pattern emerges. The distributional difference is huge—more than twice wider in log terms in the case the non-union firms invasion succeeds, which in turn produces a far more skewed income distribution and larger Gini coefficient in the economy as a whole, as shown in Table 3.

What happens to productivity growth? Figure 6(a) compares productivity growth in the two scenarios. The successful invasion of non-union firms reduces overall productivity growth median by 0.20 percentage points—a significant slowdown caused by deunionization due to the entering non-union firms having lower productivity than the union firms at the lower tail of the union distribution whose productivity advantage does not compensate for the union wage premium. Figure 6(b) shows a further result: a fall in the quality of goods due to the workers shorter tenure and skills, which maps into lower quality products. In the non-union scenario, non-union firms prevail independently of product quality as their cheaper and less skilled labor compensate for their inferior quality. The invasion, together with the productivity slowdown, entails also a deflationary tendency in long-run price consumer index (Figure 6(c)). Finally, Figure 6(d) shows that the concentration is substantially higher in the invasion case with fewer firms appropriating a higher fraction of the market. This is consistent with a more heterogeneous sales growth dynamics when invasion succeeds, with fewer firms experiencing substantial profits and more facing losses.

Figure 7 examines the timing of the deceleration and increased dispersion of productivity growth in some detail. Figure 7(a) shows that the rate of growth of productivity of the non-union firms is lower when they come to dominate the market than productivity growth of the union firms in the no-invasion case. It also reveals that the invasion scenario is not symmetric. Whereas a small niche of union firms precariously survive in the successful non-union invasion, in the latter scenario the non-union invaders eventually all die. Figure 7(b) shows that the standard deviation

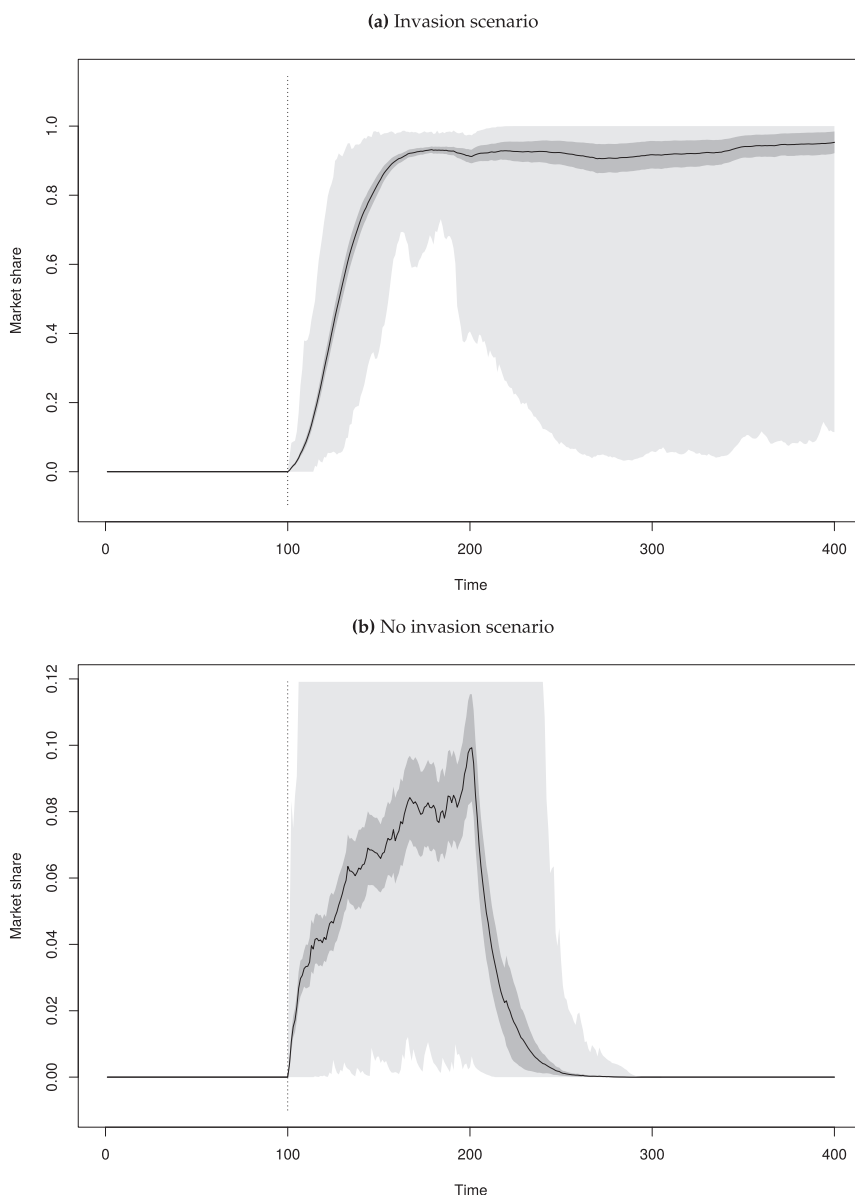


Figure 4. Market shares of non-unionized, consumer-good firms. (a) Invasion scenario and (b) no invasion scenario. Lines: average over 100 MC runs—dotted vertical line: period of starting invasion. Dark grey band: MC 95% confidence interval—light grey band: MC absolute max./min.

of productivity increases in the transient period, irrespective of the long-term outcomes, and in a successful invasion settles at a level higher than in the pre-invasion period.

What happens to other outcomes? [Table 3](#) summarizes the performance of the two scenarios in the model in terms of average values for all substantial outcomes, where for simplicity we take the invasion configuration as the baseline. All the scenario averages (from 100 Monte Carlo runs) are statistically different at 1% significance or less. Long-run GDP growth is 17.6% higher in the no-invasion case, indicating the relevance of unions to the potential output. This gap is mostly explained by the gain of 13.8% in the productivity growth when unionized firms prevail. In terms of distribution, persistent unionization is associated with a more equal distribution of wages, a smaller discretionary part of wages themselves (via bonuses) and a lower industry concentration.

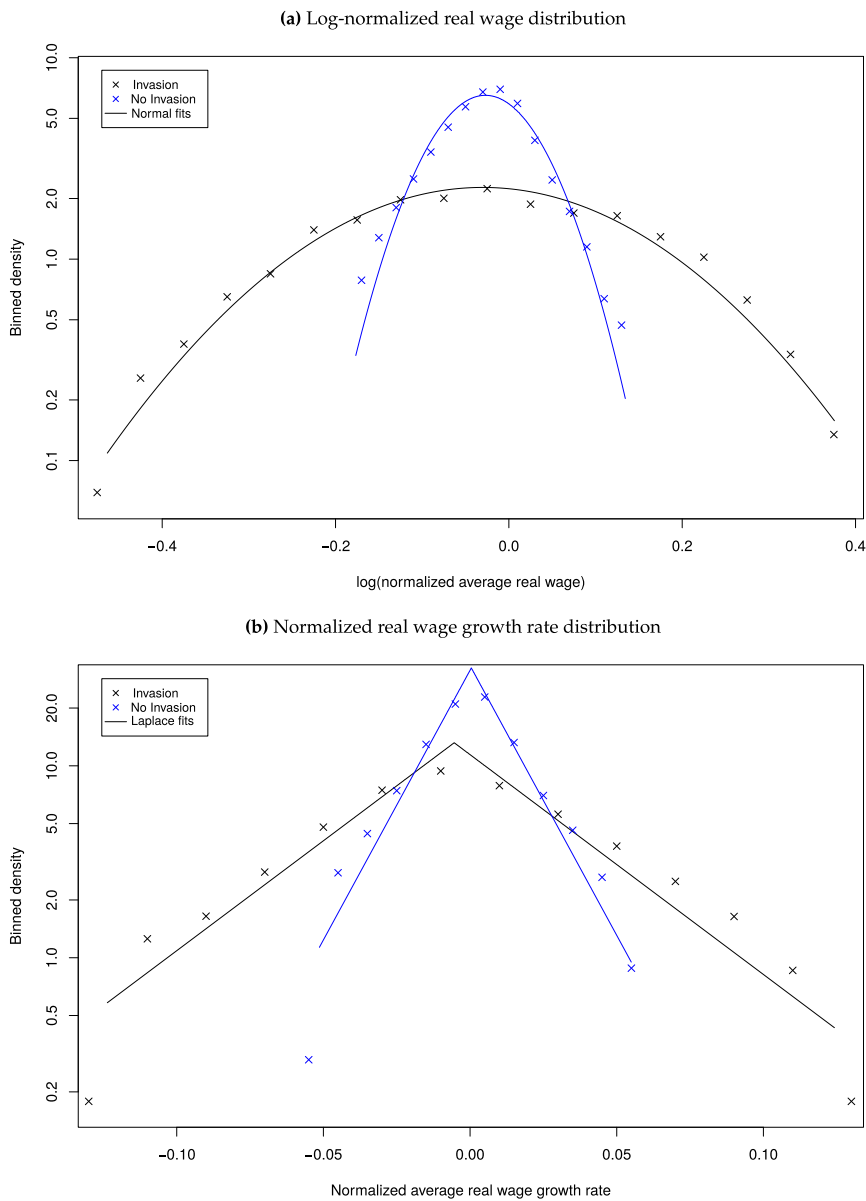


Figure 5. Comparison of wage dynamics between two scenarios (consumer-good sector). Pooled data from 100 MC runs in period [200, 400]. (a) Log-normalized real wage distribution. (b) Normalized real wage growth rate distribution.

Figure B1 in the Appendix B shows the temporal dynamics in the performance of union and non-union firms for selected outcomes. We further probed the model with a global sensitivity analysis (SA) to see how different parametrizations affect the qualitative results. Appendix C shows that the model is robust to different parametrizations. The parameters which influence collective outcomes have only marginal effects on the latter, which makes the entire model sufficiently stable.¹⁶

16 This addresses the criticism of ABM's concerning the role of "lucky" parameter choices in results. Fagiolo *et al.* (2019) discuss validation of agent-based models. Dosi *et al.* (2018) detail the SA methodology.

Table 3. Performance comparison between Invasion and No invasion scenario, selected model outcomes

	Invasion Baseline	No invasion Ratio	<i>P</i> -value
GDP growth	0.014	1.176	0.000
Productivity growth	0.014	1.138	0.000
Inflation (CPI)	−0.001	0.215	0.009
Quality index	1.535	1.118	0.000
Market concentration (HHI)	0.020	0.350	0.000
Wages standard deviation	0.223	0.501	0.000
Bonus-to-wage ratio	0.024	0.817	0.000
Gini index	0.209	0.447	0.000

Baseline values are averages over 100 MC runs in period [200, 400]. Ratios over the Invasion baseline scenario. *P*-values for a two-means *t*-test between scenarios. H_0 , no difference between scenarios.

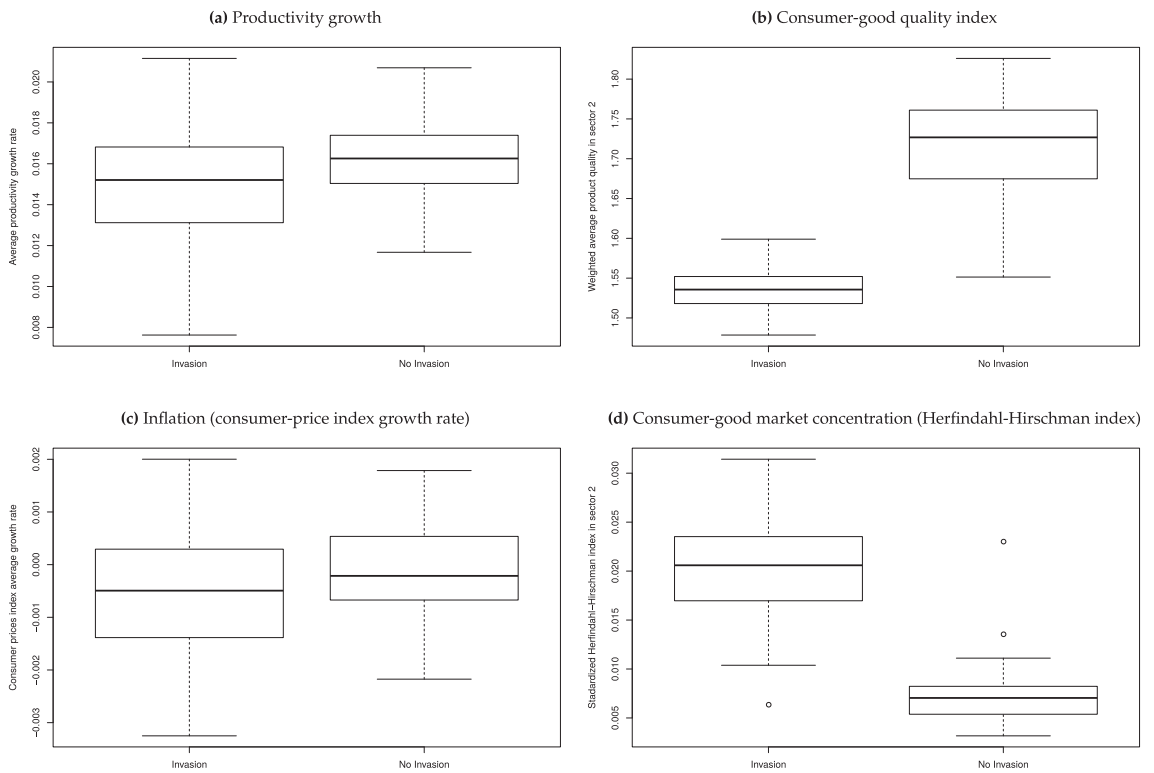


Figure 6. Performance comparison between unionized and non-unionized firms in two scenarios. (a) Productivity growth, (b) consumer-good quality index, (c) inflation (consumer-price index growth rate), and (d) consumer-good market concentration (Herfindahl-Hirschman index) Statistics for 100 MC runs in period [200, 400]. Bar: MC median—Box: MC 2nd–3rd quartile—Whiskers: MC maximum–minimum—Dots: MC outliers.

5. Shift-and-share decomposition of productivity growth

To see if within-firm adjustments due to firm-specific learning or reallocation of labor contributed most to the deceleration of productivity growth, we decomposed simulated productivity growth in the consumption-good sector into its shift and share components per [Foster *et al.* \(2001\)](#):

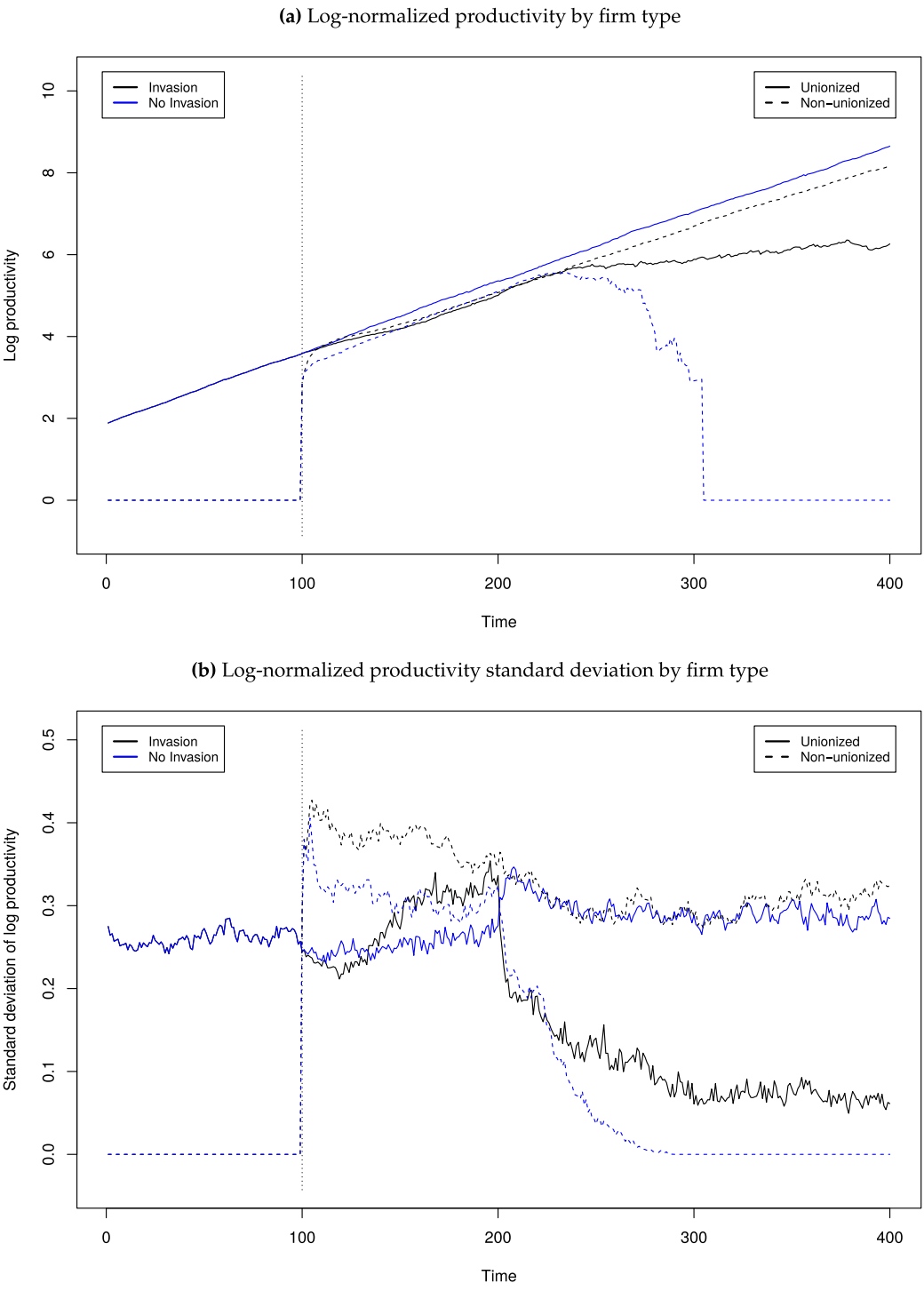


Figure 7. Comparison of productivity dynamics by firm types between two scenarios (consumer-good sector). (a) Log-normalized productivity by firm type. (b) Log-normalized productivity standard deviation by firm type. Lines: MC average over 100 MC runs—dotted vertical line: period of starting invasion.

Table 4 Shift-and-share decomposition of log-normalized labor productivity growth in two scenarios

	Total	Within	Between	Cross	Entry	Exit	Unexpl.
Invasion							
Overall	21.74 (0.64)	17.69 (0.59)	9.44 (0.72)	−6.33 (0.39)	3.66 (1.00)	−3.78 (1.13)	5.37 (0.38)
Union	−0.55 (0.84)	1.11 (0.27)	2.81 (0.57)	−0.70 (0.17)	1.24 (0.22)	−4.59 (0.60)	2.07 (0.54)
Non-union	22.29 (1.09)	16.58 (0.65)	6.63 (0.66)	−5.62 (0.31)	2.42 (0.80)	0.81 (0.60)	6.50 (0.59)
No invasion							
Overall	25.08 (0.40)	20.16 (0.50)	8.53 (0.33)	−8.82 (0.35)	0.77 (0.46)	0.74 (0.70)	3.96 (0.25)
Union	29.97 (0.61)	20.12 (0.51)	10.38 (0.37)	−8.78 (0.35)	−0.76 (0.23)	4.50 (0.34)	4.78 (0.39)
Non-union	−4.88 (0.45)	0.04 (0.02)	−1.85 (0.18)	−0.03 (0.01)	1.52 (0.25)	−3.76 (0.40)	0.07 (0.06)

Moving averages (8-period rolling window) over 100 MC runs in period [200, 400]. MC standard errors in parentheses.

$$\begin{aligned}
 \Delta \log A_t = & \underbrace{\sum_j \overbrace{f_{j,t-h} \Delta \log A_{j,t}}^{WITHIN} + \sum_j \overbrace{\Delta f_{j,t} (\log A_{j,t-h} - \log A_{t-h})}^{BETWEEN} + \sum_j \overbrace{\Delta \log A_{j,t} \Delta f_{j,t}}^{CROSS}}_{INCUMBENTS} \\
 & + \underbrace{\sum_j \overbrace{f_{j,t} (\log A_{j,t} - \log A_{t-h})}^{ENTRY}}_{ENTRY} - \underbrace{\sum_j \overbrace{f_{j,t-h} (\log A_{j,t-h} - \log A_{t-h})}^{EXIT}}_{EXIT}
 \end{aligned} \tag{12}$$

where $f_{j,t}$ is the employment share and $A_{j,t}$ is the labor productivity of firm j , and $\log A_t$ is the sectoral weighted average (log) productivity in period t . The first term is the *within-firm* component of productivity growth measured by the firm level productivity change weighted by firm's share of labor. The second term is the *between-firm* component measured by the firm change labor share weighted by the firm's relative productivity. The third term captures the covariance of the firms' productivities and labor allocations. The last two terms measure the proportional contribution of the *entry* and *exit* of firms in the market. The decomposition is computed over a rolling window of fixed length (set at 8 periods), which adds an extra term for the unexplained difference between the total and the sum of the decomposition components.

Table 4 presents the overall productivity growth for 200 periods post the initial influx of non-union firms. Figure B2(a) of Appendix B presents the decomposition results for the final part of the transient period [170; 200]. Figure B2(b) gives results for union firms and Figure B2(c) for non-union ones. The analysis of this period highlights the drivers of productivity dynamics when a significant number of both firm types still coexist. The decomposition shows that:

1. The within component reflecting the accumulation of firm capabilities and worker skills accounts for the largest part of productivity growth, though the between component has a non-negligible impact on productivity growth as well.
2. Entry and exit plays a small net role in the long run. The exit of unionized firms reduces productivity growth as deunionization proceeds.

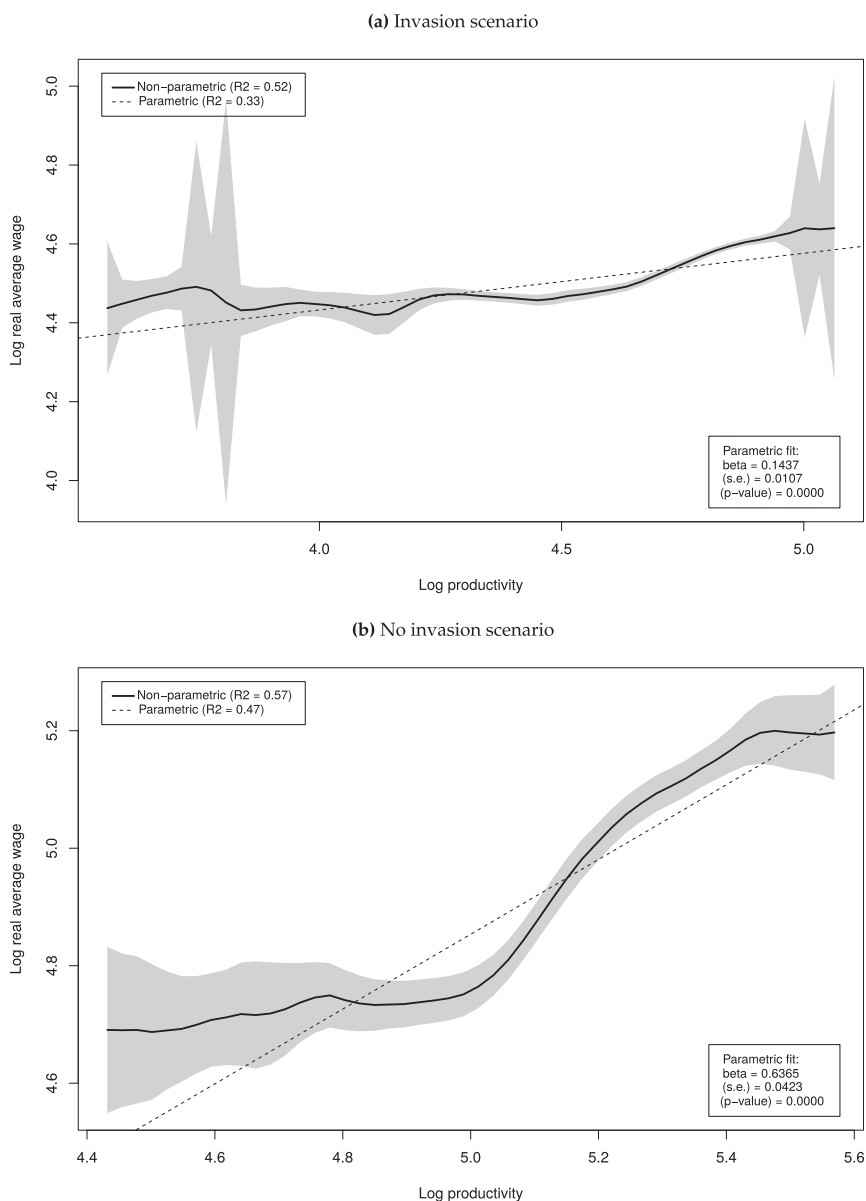


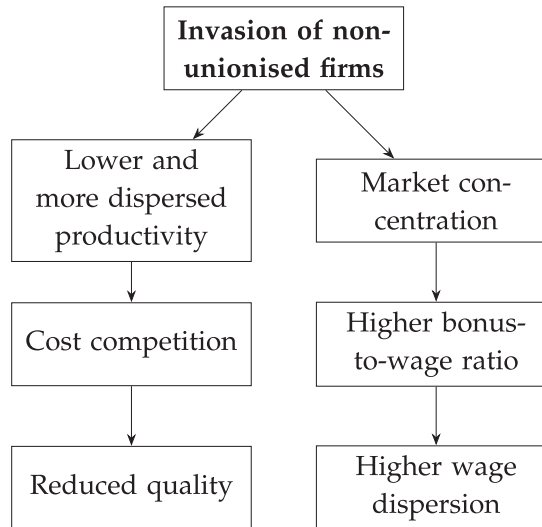
Figure 8. Correlation between real wage and productivity in two scenarios, representative runs. (a) Invasion scenario. (b) No invasion scenario Epanechnikov-kernel, non-parametric, and ordinary least squares parametric fits. Single-run data from period 200. Grey band: non-parametric fit 95% confidence interval.

3. Union firms in the no-invasion scenario exhibit substantially higher productivity growth compared to non-union firms in the invasion case. This fits with the slowdown of productivity growth from the post-World War II “golden age of capitalism” to the 1970s/1980s as compared to the 2010s period that accompanies deunionization.
4. The market selection intensity, measured by the difference between the total productivity growth and the within component, is higher in the no-invasion scenario.

Table 5. (Log) real wages vs. (log) productivity, two scenarios

	Invasion	No invasion
Intercept (α)	1.575 (0.120)	4.308 (0.098)
Slope (β)	0.634 (0.025)	0.142 (0.011)
P -value (β)	0.006 (0.004)	0.018 (0.008)
R^2 (parametric)	0.377 (0.016)	0.239 (0.018)
R^2 (non-parametric)	0.487 (0.016)	0.321 (0.021)
Observations (firms)	310.4 (4.333)	339.6 (5.020)

Ordinary least squares (parametric) and Epanechnikov-kernel (non-parametric) fits. Averages over 100 MC runs in period 200. MC standard errors in parentheses.

**Figure 9.** Dynamic path of the impacts of deunionization on outcomes.

Following [Bagger *et al.* \(2014\)](#), we regress the average real wage rw_j paid by firm j on firm productivity A_j to analyze the relation between wages and productivity in the period just after the transition to the largely non-union world ($t=200$):

$$\log rw_j = \alpha + \beta \log A_j + \epsilon_j, \quad (13)$$

where ϵ_j is the error term. We fit the equation for each Monte Carlo (MC) realization in both scenarios using OLS. The purpose is to evaluate the degree according to which more productive are also higher-wage firms. Typical outcomes are depicted in [Figure 8](#) for a representative simulation run. For comparison, a non-parametric regression is also estimated using an Epanechnikov kernel. [Table 5](#) gives the regression results for the full set of 100 MC runs. The estimated slope parameters are typically significant at the 1% level for most runs and the mean R^2 are quite high.¹⁷ The improvement associated to non-parametric estimation suggests a mildly non-linear relation between the two variables.

17 The small MC standard errors indicate that most model realizations produce results quite close to the averages.

The smaller firm-level elasticities of wages to productivities in the no-invasion scenario than in the invasion scenario means that wages growing in relatively uniform manners in the union regime favor selection among firms driven by relative efficiencies per Figure 1A above. Conversely, non-union firm wages track much more closely firm-level productivities, which tend to shelter less efficient firms from competitive selection (Figure 1B, C). Figure 9 presents the chain of feedback mechanisms occurring throughout the process.

6. Conclusions

The Agent-Based Model in this study endogenously accounts for the deunionization found in most advanced economies in the past few decades and shows that it is intrinsically related to the sluggish growth and widened dispersion among firms in productivity and wages. Starting from a fully unionized economy in which firms pay a collectively determined wage related to market productivity and face strong hiring/firing restrictions, we traced out the impacts on economic outcomes of an invasion of non-union firms that paid workers wages proportional to their individual productivity.

The outcome which fits observed phenomena is when the non-union invasion triumphs. This produces an economy with lower GDP, skills, and productivity growth, higher dispersion of wages and productivity among firms and a lower effectiveness of market competition in weeding out less efficient firms than when unionized firms maintain their market presence. While *innovative opportunities* are the same in the non-union world and in the scenario where union firms survive the invasion, absence of a collective mechanism of wage formation in the non-union setting dampens the power of efficiency-driven market selection of firms and allows the opposite selection process to emerge, where the low wages paid by the least productive/lowest skill firms drive out the most productive ones.

Viewed broadly, our results suggest that an economy in which collective wage-setting narrows the distribution of wages and institutional rules guide hiring/firing will outperform an economy in which low productivity firms can compete through low wages. In terms of growth and dispersion of pay and productivity, there is *no equity-efficiency trade-off*. Rather, the simulated model offers an explanation of the concordance of deunionization, rising dispersion of firm outcomes, and sluggish productivity growth over the past several decades. The market forces that we simulated were unable to control inequality and stagnation much as they have failed to do so in the real world. The Invisible Hand seemingly needs some strong and visible assistance in achieving equitable and efficient outcomes.

Acknowledgments

We thank for useful comments the participants to the Society for Computation in Economics and Finance (CEF), Milan (IT), 2018; WEHIA Conference, Tokyo (JP), 2018; International Schumpeter Society (ISS), Seoul (South Korea), 2018; EAEPE Conference, Nice (FR), 2018; AISRE Conference, Bolzano (IT), 2018; ANPEC Conference, Rio de Janeiro (BR), 2018; ICC-Tsinghua Conference, Beijing (CN), 2019. We gratefully acknowledge the support by the European Union's Horizon 2020 research and innovation programme under grant agreement No. 822781 GROWINPRO—Growth Welfare Innovation Productivity—and by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), Process No. 2015/24341-7.

References

- Bagger J.B. J.Christensen D. T.Mortensen (2014), 'Wage and labor productivity dispersion: The roles of total factor productivity, labor quality, capital intensity, and rent sharing,' in *2014 Meeting Papers*.
- Barth E.A.Bryson J. C.Davis R.Freeman (2016), 'It's where you work: increases in the dispersion of earnings across establishments and individuals in the United States,' *Journal of Labor Economics*, **34** (S2), S67–S97.
- Barth E.K. O.Moen F.Willumsen (2014), 'The Scandinavian model an interpretation,' *Journal of Public Economics*, **117**, 60–72.
- Berlingieri G.P.Blanchenay C.Crisuolo (2017), 'The great divergence (s),' Technical Report, OECD Science, Technology and Industry Policy Papers, No. 39, OECD Publishing, Paris.
- Bottazzi G.A.Secchi (2006), 'Explaining the distribution of firm growth rates,' *The RAND Journal of Economics*, **37** (2), 235–256.
- Bryson A.R.Freeman R.Gomez P.Willman (2017), 'The twin track model of employee voice: an Anglo-American perspective on union decline and the rise of alternative forms of voice,' Technical Report, IZA Discussion Paper No. 11223.
- Bryson A.R.Gomez M.Gunderson N.Meltz (2005), 'Youth-adult differences in the demand for unionization: are American, British, and Canadian workers all that different?,' *Journal of Labor Research*, **26** (1), 155–167.
- Calvino F.C.Crisuolo C.Menon A.Secchi (2018), 'Growth volatility and size: a firm-level study,' *Journal of Economic Dynamics and Control*, **90**, 390–407.
- Cioppa T. M.T. W.Lucas (2007), 'Efficient nearly orthogonal and space-filling Latin hypercubes,' *Technometrics*, **49** (1), 45–55.
- Decker R. A.J.Haltiwanger R. S.Jarmin J.Miranda (2016), 'Declining business dynamism: what we know and the way forward,' *American Economic Review*, **106** (5), 203–207.

- Dosi G.G. Fagiolo M. Napoletano A. Roventini T. Treibich (2015), 'Fiscal and monetary policies in complex evolving economies,' *Journal of Economic Dynamics and Control*, **52**, 166–189.
- Dosi G.G. Fagiolo A. Roventini (2010), 'Schumpeter meeting Keynes: a policy-friendly model of endogenous growth and business cycles,' *Journal of Economic Dynamics and Control*, **34** (9), 1748–1767.
- Dosi G.D. Moschella E. Pugliese F. Tamagni (2015), 'Productivity, market selection, and corporate growth: comparative evidence across US and Europe,' *Small Business Economics*, **45** (3), 643–672.
- Dosi G.M. Napoletano A. Roventini J. E. Stiglitz T. Treibich (2020), 'Rational heuristics? expectations and behaviors in evolving economies with heterogeneous interacting agents,' *Economic Inquiry*, **58**(3), 1487–1516.
- Dosi G.R. Nelson S. Winter (2001), *The Nature and Dynamics of Organizational Capabilities*. Oxford: OUP.
- Dosi G.M. C. Pereira A. Roventini M. E. Virgillito (2017), 'When more flexibility yields more fragility: the microfoundations of Keynesian aggregate unemployment,' *Journal of Economic Dynamics and Control*, **81**, 162–186.
- Dosi G.M. C. Pereira A. Roventini M. E. Virgillito (2018), 'Causes and consequences of hysteresis: aggregate demand, productivity, and employment,' *Industrial and Corporate Change*, **27** (6), 1015–1044.
- Dosi G.M. C. Pereira M. E. Virgillito (2017), 'The footprint of evolutionary processes of learning and selection upon the statistical properties of industrial dynamics,' *Industrial and Corporate Change*, **26** (2), 187–210.
- Dosi G.M. C. Pereira M. E. Virgillito (2018), 'On the robustness of the fat-tailed distribution of firm growth rates: a global sensitivity analysis,' *Journal of Economic Interaction and Coordination*, **13** (1), 173–193.
- Dunne T. L. Foster J. Haltiwanger K. R. Troske (2004), 'Wage and productivity dispersion in united states manufacturing: the role of computer investment,' *Journal of Labor Economics*, **22** (2), 397–429.
- Ebbinghaus B. J. Visser (1999), 'When institutions matter: union growth and decline in Western Europe, 1950–1995,' *European Sociological Review*, **15** (2), 135–158.
- Fagiolo G. M. Guerini F. Lamperti A. Moneta A. Roventini (2019), 'Validation of agent-based models in economics and finance,' in C. Beisbart and N. J. Saam (eds), *Computer Simulation Validation*. Springer, Cham, pp. 763–787.
- Farber H. S. D. Herbst L. Kuziemko S. Naidu (2018), 'Unions and inequality over the twentieth century: New evidence from survey data,' Technical Report, National Bureau of Economic Research.
- Foster L. C. Grim J. Haltiwanger (2016), 'Reallocation in the great recession: cleansing or not?,' *Journal of Labor Economics*, **34** (S1), S293–S331.
- Foster L. J. C. Haltiwanger C. J. Krizan (2001), 'Aggregate productivity growth: lessons from microeconomic evidence,' in *New Developments in Productivity Analysis*. University of Chicago Press, pp. 303–372.
- Freeman R. (1980), 'The exit-voice tradeoff in the labor market: unionism, job tenure, quits, and separations,' *Quarterly Journal of Economics*, **94** (4), 643–673.
- Freeman R. B. (1984), 'Longitudinal analyses of the effects of trade unions,' *Journal of Labor Economics*, **2** (1), 1–26.
- Freeman R. B. (1997), 'Spurts in union growth: defining moments and social processes,' Technical Report, National Bureau of Economic Research.
- Hibbs D. A. H. Locking Jr (2000), 'Wage dispersion and productive efficiency: evidence for Sweden,' *Journal of Labor Economics*, **18**(4), 755–782.
- Hutchinson J. D. Persyn (2012), 'Globalisation, concentration and footloose firms: in search of the main cause of the declining labour share,' *Review of World Economics*, **148**(1), 17–43.
- Karabarbounis L. B. Neiman (2014), 'The global decline of the labor share,' *The Quarterly Journal of Economics*, **129**(1), 61–103.
- McGowan M. A. D. Andrews V. Millot (2017), 'Insolvency regimes, zombie firms and capital reallocation,' Technical Report, OECD Economics Department Working Papers, No. 1399, OECD Publishing, Paris.
- Morris M. D. (1991), 'Factorial sampling plans for preliminary computational experiments,' *Technometrics*, **33** (2), 161–174.
- Saltelli A. M. Ratto T. Andres F. Campolongo J. Cariboni D. Gatelli M. Saisana S. Tarantola (2008), *Global Sensitivity Analysis: The Primer*. New York: John Wiley & Sons.
- Salter W. (1960), *Productivity & Technical Change*. Cambridge: Cambridge University Press.
- Schwellnus C. A. Kappeler P.-A. Pionnier (2017), 'Decoupling of wages from productivity,' Technical Report, OECD Economics Department Working Papers, No. 1373, OECD Publishing, Paris.
- Soboll M. (1993), 'Sensitivity analysis for non-linear mathematical models,' *Mathematical Modelling and Computational Experiments*, **1** (4), 407–414.
- Stansbury A. L. H. Summers (2020), 'The declining worker power hypothesis: an explanation for the recent evolution of the American economy,' NBER WP w27193, National Bureau of Economic Research.
- Syverson C. (2017), 'Challenges to mismeasurement explanations for the US productivity slowdown,' *Journal of Economic Perspectives*, **31**(2), 165–186.

Appendix A

Model description: technical change, investment, and entry

The technology of capital-good firms is defined as (A_i^τ, B_i^τ) . A_i^τ is the labor productivity of the machine-tool manufactured by firm i for the consumption-good sector, while B_i^τ is the labor productivity to produce the machine. Superscript τ denotes the technology vintage being produced/used. Given the monetary average wage $w_{i,t}$ paid by firm i , its unit cost of production is:

$$c_{i,t} = \frac{w_{i,t}}{B_i^\tau}. \quad (14)$$

Under a fixed mark-up $\mu_1 \in \mathbb{R}_+$ pricing rule, price $p_{i,t}$ of firm i is defined as:

$$p_{i,t} = (1 + \mu_1)c_{i,t}. \quad (15)$$

Firms in the capital-good industry adaptively strive to increase market shares and profits by improving technology via innovation and imitation. Firms invest in R&D a fraction $\nu \in [0, 1]$ of their past sales $S_{i,t-1}$:

$$RD_{i,t} = \nu S_{i,t-1}. \quad (16)$$

R&D activity is performed by workers devoted to this activity, whose demand is:

$$L_{i,t}^{R\&D} = \frac{RD_{i,t}}{w_{i,t}}. \quad (17)$$

Firms split their R&D workers $L_{i,t}^{R\&D}$ between innovation ($IN_{i,t}$) and imitation ($IM_{i,t}$) activities according to the parameter $\zeta \in [0, 1]$:

$$IN_{i,t} = \zeta L_{i,t}^{R\&D}, \quad (18)$$

$$IM_{i,t} = (1 - \zeta)L_{i,t}^{R\&D}. \quad (19)$$

Innovation is a two-step process. The first determines whether a firm obtains or not access to an innovation—irrespective of whether it will ultimately be a success or a failure—through a draw from a Bernoulli distribution with mean:

$$\theta_{i,t}^{\text{in}} = 1 - e^{-\zeta_1 IN_{i,t}}, \quad (20)$$

with parameter $\zeta_1 \in [0, 1]$. If a firm innovates, it may draw a new machine-embodying technology $(A_{i,t}^{\text{in}}, B_{i,t}^{\text{in}})$ according to:

$$A_{i,t}^{\text{in}} = A_{i,t}(1 + x_{i,t}^A), \quad (21)$$

$$B_{i,t}^{\text{in}} = B_{i,t}(1 + x_{i,t}^B), \quad (22)$$

where $x_{i,t}^A$ and $x_{i,t}^B$ are two independent draws from a Beta(α_1, β_1) distribution, $(\alpha_1, \beta_1) \in \mathbb{R}_+^2$ over the fixed support $[\underline{x}_1, \bar{x}_1] \subset \mathbb{R}$.

Imitation also follows a two-step procedure. The access to imitation come from sampling a Bernoulli with mean:

$$\theta_{i,t}^{\text{im}} = 1 - e^{-\zeta_2 IM_{i,t}}, \quad (23)$$

being parameter $\zeta_2 \in [0, 1]$. Firms accessing the second stage may copy technology $(A_i^{\text{im}}, B_i^{\text{im}})$ from a close competitor and select the machine to produce using the rule:

$$\min[p_{i,t}^b + bc_{A_i^b, j,t}^b], \quad b = \tau, \text{in}, \text{im}, \quad (24)$$

where $b \in \mathbb{R}_+$ is a payback parameter.

Firms in consumption-good sector do not conduct R&D, instead they access new technologies incorporating new machines to their existing capital stock $\Xi_{j,t}$. Firms invest according to expected demand $D_{j,t}^e$, computed by an adaptive rule:

$$D_{j,t}^e = g(D_{j,t-1}, D_{j,t-2}, D_{j,t-h}), \quad 0 < h < t, \quad (25)$$

where $D_{j,t-h}$ is the actual demand faced by firm j at time $t - h$. $h \in \mathbb{N}_*$ is a parameter and $g: \mathbb{R}^h \rightarrow \mathbb{R}_+$ is the expectation function, usually an unweighted moving average over four periods. The corresponding desired level of production $Q_{j,t}^d$, considering the actual inventories $N_{j,t}$ from previous period, is:

$$Q_{j,t}^d = (1 + \iota) D_{j,t}^e - N_{j,t-1}, \quad (26)$$

being $N_{j,t}^d = \iota D_{j,t}^e$ the desired inventories and $\iota \in \mathbb{R}_+$ a parameter.

If the desired capital stock K_j^d —computed as a linear function of the desired level of production $Q_{j,t}^d$ —is higher than the current $K_{j,t}$, firms invest $El_{j,t}^d$ to expand capacity:

$$El_{j,t}^d = K_{j,t}^d - K_{j,t-1}. \quad (27)$$

Replacement investment $Sl_{j,t}^d$, to substitute a set $RS_{j,t}$ of existing machines by more productive ones, is decided according to a fixed payback period $b \in \mathbb{R}_+$. Machines $A_i^z \in \Xi_{j,t}$ are evaluated by the ratio between the price of new machines and the corresponding cost savings:

$$RS_{j,t} = \left\{ A_i^z \in \Xi_{j,t} : \frac{p_{i,t}^*}{c_{j,t}^{A_i^z} - c_{j,t}^*} \leq b \right\}, \quad (28)$$

where $p_{i,t}^*$ and $c_{j,t}^*$ are the price and unit cost of production upon the selected new machine.

Prospective firms in both sectors decide on entry based on the number F_{t-1}^z ($z = 1, 2$) and financial conditions of incumbents. The number of entrants in sector z is:

$$b_t^z = \max[(o\pi_t^z + (1 - o)MA_t^z)F_{t-1}^z, 0], \quad z = 1, 2, \quad (29)$$

being $o \in [0, 1]$ a mix parameter and π_t^z a uniform random draw on the fixed support $[\underline{x}_2^z, \bar{x}_2^z]$ representing the idiosyncratic component in the entry process. The sectoral market attractiveness MA_t^z is evaluated based on the dynamics of firms' balance sheets:

$$MA_t^z = MC_t^z - MC_{t-1}^z \quad (\text{bounded to } [\underline{x}_2^z, \bar{x}_2^z]), \quad (30)$$

defined as the (log) ratio between the aggregate sectoral stocks of liquid assets NW_{t-1}^z (bank deposits) and debt Deb_{t-1}^z (bank loans):

$$MC_t^z = \log NW_{t-1}^z - \log Deb_{t-1}^z. \quad (31)$$

Appendix A: Labor search-and-match

Labor demand in the consumption-good sector $L_{j,t}^d$ is determined by desired production $Q_{j,t}^d$ and the average productivity of current capital stock $A_{j,t}$:

$$L_{j,t}^d = \frac{Q_{j,t}^d}{A_{j,t}}. \quad (32)$$

In the capital-good sector, instead, $L_{i,t}^d$ considers orders $Q_{i,t}$ and labor productivity $B_{i,t}$. In what follows, only the behavior of the consumption-good firms (subscript j) is shown but capital-good sector operate under the same rules, except it follows the wage offers from top-paying firms in the consumption-good sector.

Firms decide whether to hire (or fire) workers according to the expected production $Q_{j,t}^d$. If it is increasing, $\Delta L_{j,t}^d$ new workers are (tentatively) hired in addition to the existing number $L_{j,t-1}$. Each firm (expectedly) get a fraction of the number of applicant workers $L_{a,t}$ in its candidates queue $\{\ell_{j,t}^s\}$, proportional to firm market share $f_{j,t-1}$:

$$E(I_{j,t}^s) = (\omega(1 - U_{t-1}) + \omega_u U_{t-1}) L_{fj,t-1}^s, \quad (33)$$

where L^s is the (fixed) total labor supply, U_t is the unemployment rate and $\omega, \omega_u \in \mathbb{R}_+$ are parameters defining the number of applications each job seeker sends if employed or unemployed, respectively. Considering the set of workers in $\{\ell_{j,t}^s\}$, each firm select the subset of desired workers $\{\ell_{j,t}^d\}$ to make a job (wage) offer:

$$\{\ell_{j,t}^d\} = \{\ell_{j,t} \in \{\ell_{j,t}^s\} : w_{\ell,t}^r \leq w_{j,t}^o\}. \quad (34)$$

Firms target workers that would accept the wage offer $w_{j,t}^o$, considering the wage $w_{\ell,t}^r$ requested by workers, if any. Firm j hires up to the total demand $L_{j,t}^d$ or up to all workers in the queue, whichever is lower. The total number of workers $L_{j,t}$ the firm will employ in t , given the current workforce $L_{j,t-1}$, is bound by:

$$0 \leq L_{j,t} \leq L_{j,t}^d \leq L_{j,t}^s, \quad L_{j,t}^z = L_{j,t-1} + \{\ell_{j,t}^z\}, \quad z = d, s. \quad (35)$$

The search, wage determination and firing processes differ according to the configuration. When there is no negotiation, firm j offers the wage:

$$w_{j,t}^o = w_{j,t-1}^o (1 + \text{WP}_{j,t} + \text{N}(0, w_{\text{err}}^o)) \quad \text{bounded to} \quad p_{j,t-1} A_{j,t-1}, \quad (36)$$

that is accepted by the worker if she has no better offer. The wage premium is defined as:

$$\text{WP}_{j,t} = \psi_2 \frac{\Delta A_t}{A_{t-1}} + \psi_4 \frac{\Delta A_{j,t}}{A_{j,t-1}}, \quad \psi_1 + \psi_2 \leq 1, \quad (37)$$

being A_t the aggregate labor productivity, Δ the time difference operator, and $(\psi_1, \psi_2) \in \mathbb{R}_+^2$ parameters. $w_{j,t}^o$ is also applied to existing workers. $w_{j,t}^o$ is bounded to the break-even wage (zero unit profits myopic expectation). When one-round of negotiation exists, workers have reservation wages equal to the unemployment benefit, if any, and request a wage $w_{\ell,t}^r$ in the job application:

$$w_{\ell,t}^r = \begin{cases} w_{\ell,t-1} (1 + \epsilon) & \text{if employed in } t-1 \\ w_{\ell,t}^s & \text{if unemployed in } t-1 \end{cases}. \quad (38)$$

$w_{\ell,t}$ is the current wage for the employed workers and $\epsilon \in \mathbb{R}_+$, a parameter. Unemployed workers have a shrinking satisfying wage $w_{\ell,t}^s$, accounting for the wage history:

$$w_{\ell,t}^s = \max \left(w_t^\mu, \frac{1}{T_s} \sum_{b=1}^{T_s} w_{\ell,t-b} \right), \quad (39)$$

being $T_s \in \mathbb{N}_*$, the moving average time-span parameter. An employed worker accepts the best offer $w_{j,t}^o$ she receives if higher than current wage $w_{\ell,t}$. An unemployed worker accepts the best offer if at least equal to the unemployment benefit w_t^μ . Government imposes a minimum wage $w_t^{\min}$ on firms, indexed on aggregate productivity A_t :

$$w_t^{\min} = w_{t-1}^{\min} (1 + \psi_1 \frac{\Delta A_t}{A_{t-1}}). \quad (40)$$

Appendix A Banks, government, and consumption

There are B commercial banks (subscript k) which take deposits and provide credit to firms. Bank-firm pairs are set randomly and are stable along firms' lifetime. Bank profits come from interest received on loans ($\text{Loans}_{k,t}$) and on reserves at the central bank ($\text{Res}_{k,t}$) deducted from interest paid on deposits ($\text{Depo}_{k,t}$) and from losses from defaulted loans ($\text{BadDeb}_{k,t}$):

$$\Pi_{k,t}^b = r^{\text{deb}} \text{Loans}_{k,t} + r^{\text{res}} \text{Res}_{k,t} - r_D \text{Depo}_{k,t} - \text{BadDeb}_{k,t}, \quad (41)$$

being $(r^{\text{deb}}, r, r_D) \in \mathbb{R}_+^3$ the interest rates on debt, bank reserves, and deposits, respectively.

Government taxes firms and banks profits at a fixed rate $tr \in \mathbb{R}_+$:

$$\text{Tax}_t = (\Pi_t^1 + \Pi_t^2 + \Pi_t^b)tr, \quad (42)$$

where Π_t^1 , Π_t^2 and Π_t^b are the aggregate total profits of the capital-good, the consumer-good and the banking sectors, respectively. It pays to unemployed workers a benefit w_t^u which is a fraction of the current average wage $\bar{w}_t$:

$$w_t^u = \psi \bar{w}_{t-1}, \quad (43)$$

where $\psi \in [0, 1]$ is a parameter. The recurring total public expenditure G_t and the public primary deficit (or surplus) are:

$$G_t = (L^S - L_t^D)w_t^u. \quad (44)$$

$$\text{Def}_t = G_t - \text{Tax}_t, \quad (45)$$

The stock of public debt is updated as in:

$$\text{Deb}_t = \text{Deb}_{t-1} + \text{Def}_t - \Pi_t^{cb} + G_t^{\text{bail}}, \quad (46)$$

where Π_t^{cb} is the operational result (profits/losses) of the central bank and G_t^{bail} is the cost of rescuing (bail-out) the banking sector during financial crises, if any.

Workers fully consume their income (if possible) and do not get credit. Accordingly, desired aggregate consumption C_t^d depends on the income of both employed and unemployed workers plus the desired unsatisfied consumption from previous periods:

$$C_t^d = \sum_{\ell} w_{\ell,t} + G_t + (C_{t-1}^d - C_{t-1}). \quad (47)$$

The effective consumption C_t is bound by the real production Q_t^2 of the consumption-good sector:

$$C_t = \min(C_t^d, Q_t^2), \quad Q_t^2 = \sum_j Q_{j,t}. \quad (48)$$

The model applies the standard national account identities by the aggregation of agents' stocks and flows. The aggregate value added by capital- and consumption-good firms Y_t equals their aggregated production Q_t^1 and Q_t^2 , respectively (there are no intermediate goods). That is equal to the sum of the effective consumption C_t , the total investment I_t and the change in firm's inventories ΔN_t :

$$Q_t^1 + Q_t^2 = Y_t = C_t + I_t + \Delta N_t. \quad (49)$$

For further details, see Dosi *et al.* (2010, 2015, 2017).

Appendix B

Appendix B Further model results

This Appendix presents graphs that show the pattern of change in several outcome variables beyond those stressed in the text.

The temporal dynamics of the HH index of market concentration is shown in Figure B1(a). Market concentration increases in both scenarios as non-unionized firms enter. After the transient ($t > 200$), the index stabilizes at marginally higher levels when the invasion fails and at much higher ones when it succeeds. Figure B1(b) shows that the invasion scenario presents significantly higher total distributed bonuses, mostly driven by the elevated profits of the few top performing firms. Figure B1(c) shows that higher bonuses and lower wages result into a higher income inequality. This contributes to a more skewed wage distribution as measured by the wage standard deviation in Figure B1(d).

Figure B2 presents the decomposition results for the shorter time lapse [170; 200] at the end of the transient period. Figure B2(a) includes the entire market while Figure B2(b) shows the contribution of unionized firms and Figure B2(c) shows the contribution of non-unionized firms, respectively.

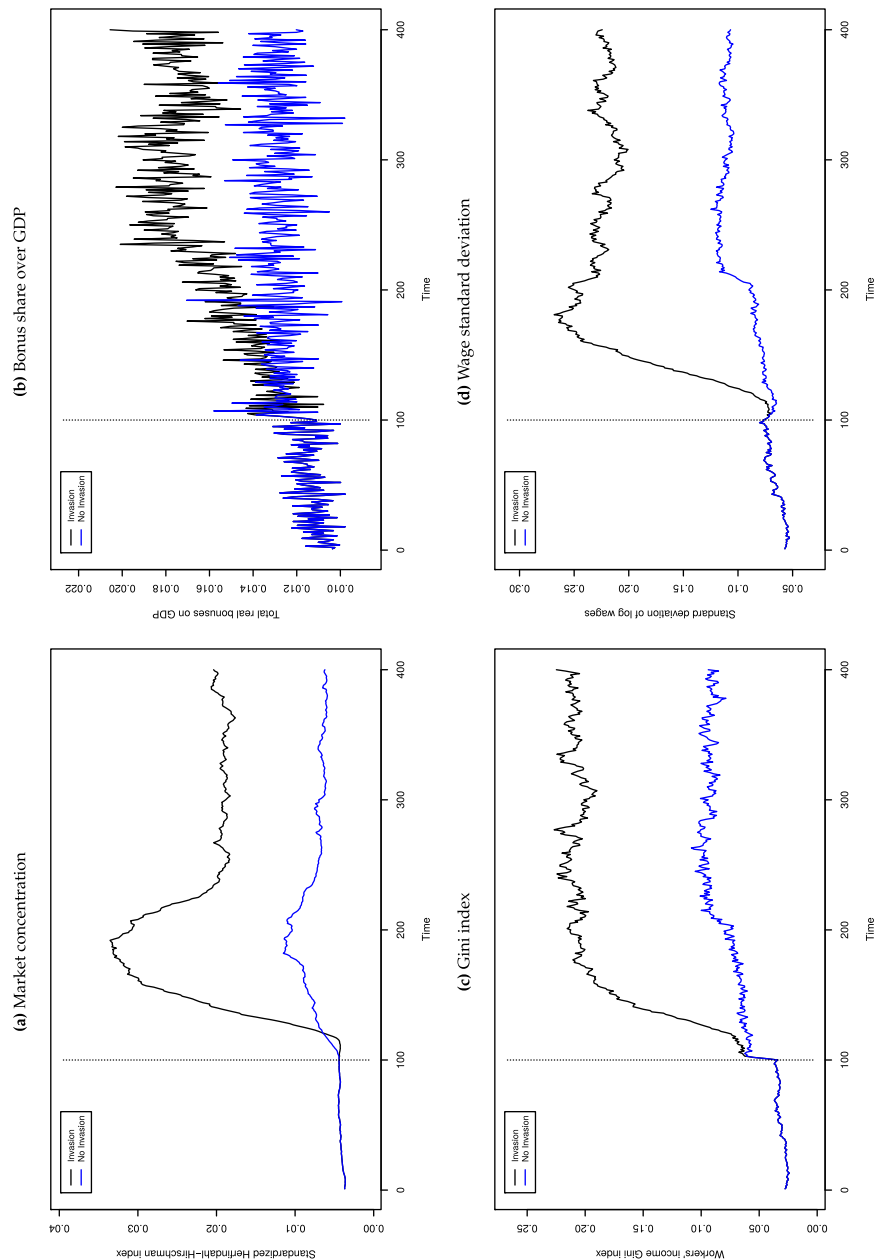


Figure B1. Comparison of market concentration (consumer-good sector) and income distribution dynamics between two scenarios. (a) Market concentration, (b) bonus share over GDP, (c) Gini index, and (d) wage standard deviation Lines: MC average over 100 MC runs—dotted vertical line: period of starting invasion.

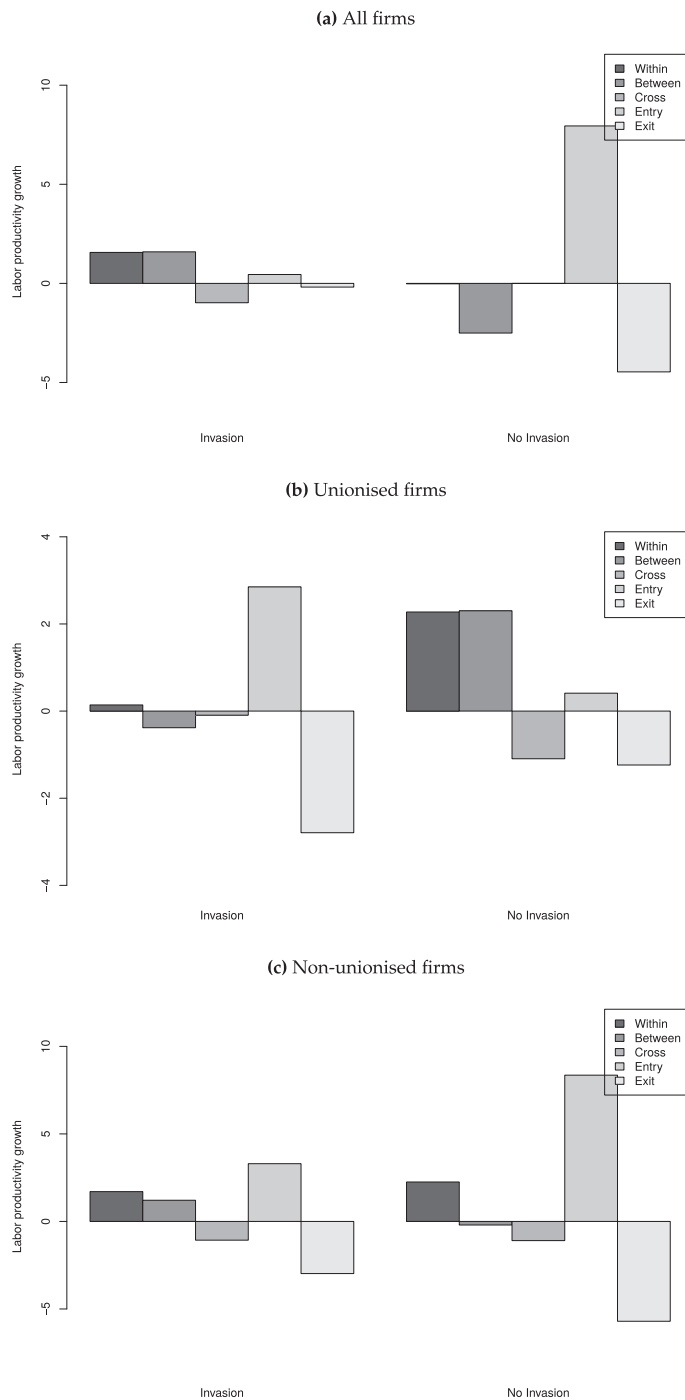


Figure B2. Comparison of FHK decomposition during the transition period in two scenarios. (a) All firms, (b) unionized firms, (c) non-unionized firms. Labor productivity growth moving averages (8-period rolling window) over 100 MC runs in period [170, 200].

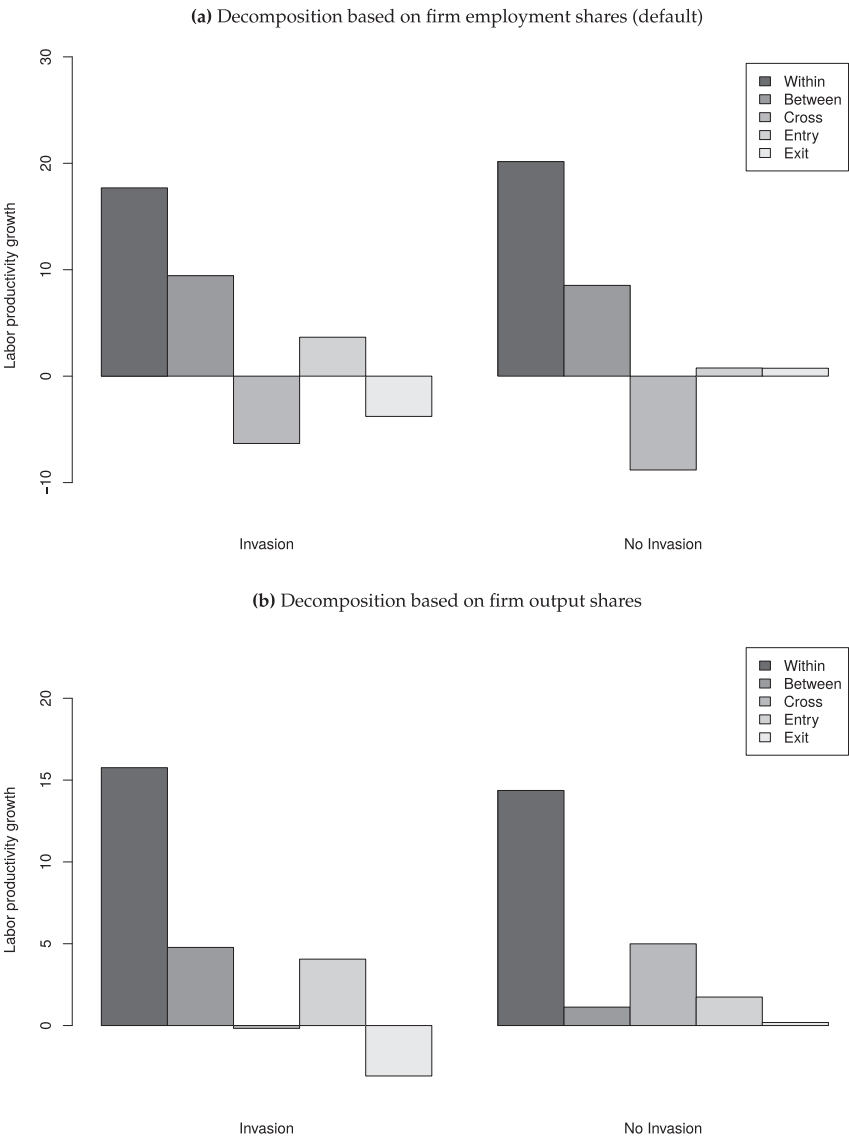


Figure B3 FHK decomposition based on different firm share measures in two scenarios. (a) Decomposition based on firm employment shares (default), (b) decomposition based on firm output shares. Labor productivity growth moving averages (8-period rolling window) over 100 MC runs in period [200; 400].

Additionally, [Figure B3](#) displays two specifications for the shift-and-share decomposition over the entire post-transient period, [Figure B3\(a\)](#) follows the evidence in [Table 4](#) with employment shares while [Figure B3\(b\)](#) applies output shares to the weights. The within-firm change dominates in both cases.

Appendix C

Appendix C Global sensitivity analysis

The K + S model is calibrated using the values presented in [Table C1](#) below (column VALUE) for the parameters and initial conditions. Global sensitivity analysis (SA) is performed across the entire parametric space, inside the closed region defined by [Table C1](#) (columns MIN. and MAX.), and the synthetic results are reported (columns μ^* , DIRECT and INTERACTION) for the most sensitive among the tested output variables (results for the remaining variables are available upon request). Two SA methodologies are employed, elementary effects (EE) and Sobol variance decomposition (SVD). The sensitivity analysis is performed on both scenarios, under successful invasion or not. However, the main results hold irrespective of the set-up.

The SA is performed for $t \in [200, 400]$, i.e., after the transient period, on a set of output variables (the “metrics”) relevant to the current discussion, namely the average overall productivity growth rate ($\Delta\bar{A}$) and standard deviation $\bar{A}^{sd}$, the market share of non-unionized firms ($\bar{f}^n$), and the turbulence in the consumption-good market measured by the average number of exiting firms ($\bar{n}^{ext,2}$).¹⁸ All the model’s parameters and initial conditions, their calibration values, as well as the key SA tests statistics, are detailed in the following.

As a first step, we apply the Morris elementary effects (EE) method on the 67 parameters and initial conditions (the “factors”). The EE analysis ([Morris, 1991](#); [Saltelli *et al.*, 2008](#)) is summarized by the μ^* statistic in [Table C1](#), which is a measure of the direct absolute effect of each factor (parameter or initial condition) on the chosen output variable, being the parametric space rescaled to the $[0, 1]$ interval on each dimension. This allows to identify those factors which significantly affect the selected model metrics. The statistical significance of this statistic, the probability of not rejecting $H_0 : \mu_i^* = 0$, is also evaluated and indicated by the usual asterisk convention. The EE computation is performed directly over model samples from an optimized 10-trajectory one-at-a-time design of experiments (DoE). Each DoE point is sampled three times to account for stochastic components. The EE analysis indicates that $\bar{n}^{ext,2}$ is the most sensitive metric, with 19 relevant factors, while $\Delta\bar{A}$ is the least sensitive, with 7 influential factors.¹⁹ $\bar{A}^{sd}$ and $\bar{f}^n$ are affected by 14 and 12 influential factors, respectively. In total, 25 unique *relevant* factors were identified after discarding duplicates.

In order to quantify the effect of each of the relevant factor over the selected metrics, directly or in interaction with other factors, as a second step we perform a Sobol Variance Decomposition (SVD). The SVD analysis is reported in [Table C1](#) by two statistics: the decomposition of the direct influence of each factor on the variance of the tested output variable, adding up to 1, (DIRECT column), and its indirect influence share, by interacting with other factors, non-linear/non-additive effects, (INTERACTION column). Each DoE point is sampled 10 times ([Sobol, 1993](#); [Saltelli *et al.*, 2008](#)). Because of the high computational cost to produce the SVD using the original simulation model, a simplified version of it—a meta-model—is estimated using the Kriging method and employed for the SVD. The meta-model is estimated by numerical maximum likelihood using a set of observations multi-sampled from the original model using a high-efficiency, nearly orthogonal Latin hypercube design of experiments ([Cioppa and Lucas, 2007](#)).

The SVD results ([Table C1](#)) indicate a smaller subset of 9 *important* factors for the chosen metrics.²⁰ These factors, in order of importance, define (i) the intensity of the competition in the consumer-good market (χ), (ii) the tenure skill-accumulation rate for employed workers (τ_T), (iii) the work-life time before retirement (T_r), (iv) the maximum technical advantage of an entrant firm (α_5), (v) the shape of the technological opportunity space for entrants (α_2, β_2), (vi) the importance of unfilled demand for firm competitiveness (ω_2), (vii) the lower bound of the entrant-firm size distribution ($\bar{x}_2^2$), and (viii) the initial number of firms in the consumer-good sector (F_0^2). The equations and values related to each parameter are presented in Appendices A and C, respectively.

- 18 Other relevant metrics, like the macro aggregates’ growth rates, the inequality measures, and the industrial performance indicators were already evaluated in previous papers based on the labor-augmented K + S model and are not be replicated here. The general results from these past analyses indicate a relatively small dependence of the model qualitative results on the chosen parametrization.
- 19 The selection criteria is to consider the top 80% EE contributors at 5% significance.
- 20 The selection criteria is to consider the top 80% SVD contributors.

Table C1 Model parameters and initial conditions, calibration values, minimum-maximum range for sensitivity analysis, elementary effects μ^* statistic ($n = 2040$ samples) and Sobol decomposition direct and interaction effects indexes ($n = 5120$ samples)

Symbols	Description	Value	Min.	Max.	μ^*	Direct	Interaction
Policy and credit market							
ϕ	Unemployment subsidy rate on average wage	0.200	0.000	1.000	0.679*	0.0191	0.00062
tr	Tax rate	0.100	0.000	0.300	0.064	0.0001	0.00058
r	Prime interest rate	0.010	0.005	0.050	0.386	0.0013	0.00058
r_D	Interest rate on bank deposits	0.000	0.000	0.010	0.189*	0.0013	0.00060
μ_{deb}	Mark-up of interest on debt over prime rate	0.000	0.100	0.500	0.124*	0.0018	0.00057
μ_{res}	Mark-up of interest on reserve to prime rate	1.000	0.500	1.000	0.086*	0.0004	0.00056
Λ	Prudential limit on debt (sales multiple)	3	1	4	0.037**	0.0008	0.00056
Λ_{min}	Prudential limit on debt (fixed floor)	20000	0	100000	0.119*	0.0008	0.00056
Labor market							
w_{err}^0	SD of error when evaluating the market wage	0.100	0.000	1.000	0.170	0.0003	0.00056
ϵ	Minimum desired wage increase rate	0.020	0.005	0.200	0.099*	0.0000	0.00055
τ_T	Skills accumulation rate on tenure	0.010	0.001	0.100	0.835*	0.2458	0.00087
τ_U	Skills deterioration rate on unemployment	0.010	0.001	0.100	0.107	0.0025	0.00054
T_r	Number of periods before retirement (work life)	120	60	240	0.252	0.0220	0.00057
T_s	Number of wage memory periods	4	1	8	0.141*	0.0034	0.00056
ω	Number of firms to send applications (employed)	5	1	20	0.217	0.0002	0.00057
ω_{un}	Number of firms to send applications (unempl.)	10	1	20	0.191	0.0034	0.00058
ψ_2	Aggregate productivity pass-trough	1.000	0.950	1.050	0.244	0.0023	0.00058
ψ_4	Firm-level productivity pass-trough	0.500	0.000	1.000	0.050*	0.0000	0.00057
ψ_6	Share of firm free cash flow paid as bonus	0.200	0.000	0.500	0.089	0.0017	0.00056
Technology							
η	Maximum machine-tools useful life	19	10	40	0.061*	0.0010	0.00058
ν	R&D investment propensity over sales	0.040	0.010	0.200	0.060*	0.0011	0.00058
ξ	Share of R&D expenditure in imitation	0.500	0.200	0.800	0.149*	0.0120	0.00063
b	Payback period for machine replacement	9	1	10	0.130	0.0004	0.00060
$\dim_{mach}$	Machine-tool unit production capacity	40	10	100	0.183	0.0003	0.00059
(α_1, β_1)	Beta distribution parameters (innovation process)	(3,3)	(1,1)	(5,5)	(0.236,0.054*)	(0.0002,0.0008)	(0.00059,0.00060)
(α_2, β_2)	Beta distribution parameters (entrant productivity)	(2,4)	(1,1)	(5,5)	(0.416***,0.584*)	(0.2949,0.1714)	(0.00113,0.00088)
(ζ_1, ζ_2)	Search capabilities for innovation/imitation	(0.300,0.300)	(0.100,0.100)	(0.600,0.600)	(0.090*,0.102*)	(0.0033,0.0095)	(0.00087,0.00085)
$[\underline{x}_1, \bar{x}_1]$	Beta distribution support (innovation process)	[-0.150,0.150]	[-0.300,0.100]	[-0.100,0.300]	(0.064*,0.308*)	(0.0002,0.0042)	(0.00088,0.00086)

Baseline values for non-unionised firms. Sensitivity analysis statistics relative to the standard deviation of firm log productivity (the most sensitive variable). μ^* statistic estimated using factors rescaled to [0, 1]. μ^* significance: *** 0.1%—** 1%—* 5%—no asterisk) not significant at 5% level.

Symbol	Description	Value	Min.	Max.	μ^*	Direct	Interaction
Industrial dynamics							
γ	Share of new customers for capital-good firm	0.500	0.200	0.800	0.175*	0.0022	0.00087
t	Desired inventories share	0.100	0.000	0.300	0.338	0.0000	0.00085
μ_1	Mark-up in capital-good sector	0.100	0.010	0.200	0.084*	0.0037	0.00086
α	Weight of market conditions for entry decision	0.500	0.000	1.000	0.060*	0.0003	0.00085
χ	Replicator dynamics coefficient (compet. intensity)	1.000	0.200	5.000	0.181*	0.0001	0.00085
ν	Mark-up adjustment coefficient	0.040	0.010	0.100	0.071	0.0001	0.00085
u	Planned utilization by consumption-good entrant	0.750	0.500	1.000	0.943	0.0064	0.00086
x_5	Max technical advantage of capital-good entrant	0.300	0.000	1.000	0.730*	0.1423	0.00109
exit ₁	Min orders to stay in capital-good sector	1	1	5	0.110	0.0111	0.00101
exit ₂	Min share to stay in consumption-good sector	10^{-5}	10^{-6}	10^{-3}	0.106	0.0001	0.00102
$[\Phi_1, \Phi_2]$	Min/max capital ratio for consumer-good entrant	[0.100, 0.900]	[0.000, 0.500]	[0.500, 1.000]	(0.266, 0.285)	(0.0003, 0.0059)	(0.00103, 0.00111)
$[\Phi_3, \Phi_4]$	Min/max net wealth ratio for capital-good entrant	[0.100, 0.900]	[0.000, 0.500]	[0.500, 1.000]	(0.076, 0.234*)	(0.0136, 0.0007)	(0.00124, 0.00115)
ω_1	Competitiveness weight for price	1.000	0.200	5.000	0.198	0.0001	0.00118
ω_2	Competitiveness weight for unfilled demand	1.000	0.200	5.000	0.338*	0.0010	0.00119
ω_3	Competitiveness weight for quality	1.000	0.200	5.000	0.133*	0.0274	0.00129
$[\bar{x}_1^1, \bar{x}_1^2]$	Entry distribution support for capital-good firm	[-0.150, 0.150]	[-0.300, 0.100]	[-0.100, 0.300]	(0.281, 0.245)	(0.0017, 0.0161)	(0.00124, 0.00124)
$[\bar{x}_2^1, \bar{x}_2^2]$	Entry distribution support for consumer-good firm	[-0.150, 0.150]	[-0.300, 0.100]	[-0.1, 0.3]	(0.232, 0.097*)	(0.0000, 0.0080)	(0.00123, 0.00122)
$[F_{\min}^1, F_{\max}^1]$	Min/max number of capital-good firms	[1, 100]	[10, 20]	[20, 40]	(0.081, 0.333)	(0.0176, 0.0040)	(0.00124, 0.00119)
$[F_{\min}^2, F_{\max}^2]$	Min/max number of consumer-good firms	[1, 400]	[100, 200]	[200, 400]	(0.072*, 0.558)	(0.0028, 0.0021)	(0.00119, 0.00118)
Initial conditions							
μ_0^2	Initial mark-up in consumption-good sector	0.200	0.100	0.500	0.220*	0.0023	0.00122
K_0	Initial capital stock in consumer-good sector	800	200	3000	0.100*	0.0005	0.00113
L_0^S	Number of workers	2.510^5	1.310^5	5.010^5	0.188	0.0042	0.00111
Sav ₀	Initial consumer unfilled-demand savings	1.110^6	0.510^6	5.010^6	0.062	0.0003	0.00113
B_0	Number of banks	10	5	15	0.203	0.0041	0.00119
(F_0^1, F_0^2)	Initial number of capital/consumption-good firms	(20, 200)	(10, 100)	(40, 400)	(0.142*, 0.198)	(0.0474, 0.0098)	(0.00131, 0.00119)
NW ₀ ⁰	Initial net wealth of banking sector	1.010^6	0.510^6	5.010^6	0.056*	0.0065	0.00118
(NW ₀ ¹ , NW ₀ ²)	Initial net wealth capital/consumption-good sector	(10,000, 5000)	(2000, 2000)	(50,000, 50,000)	(0.471, 0.259)	(0.0072, 0.0007)	(0.00111, 0.00106)

Baseline values for non-uniformed firms. Sensitivity analysis statistics relative to the standard deviation of firm log productivity (the most sensitive variable). μ^* statistic estimated using factors rescaled to [0, 1]. μ^* significance: *** 0.1%—** 1%—* 5%—(no asterisk) not significant at 5% level.

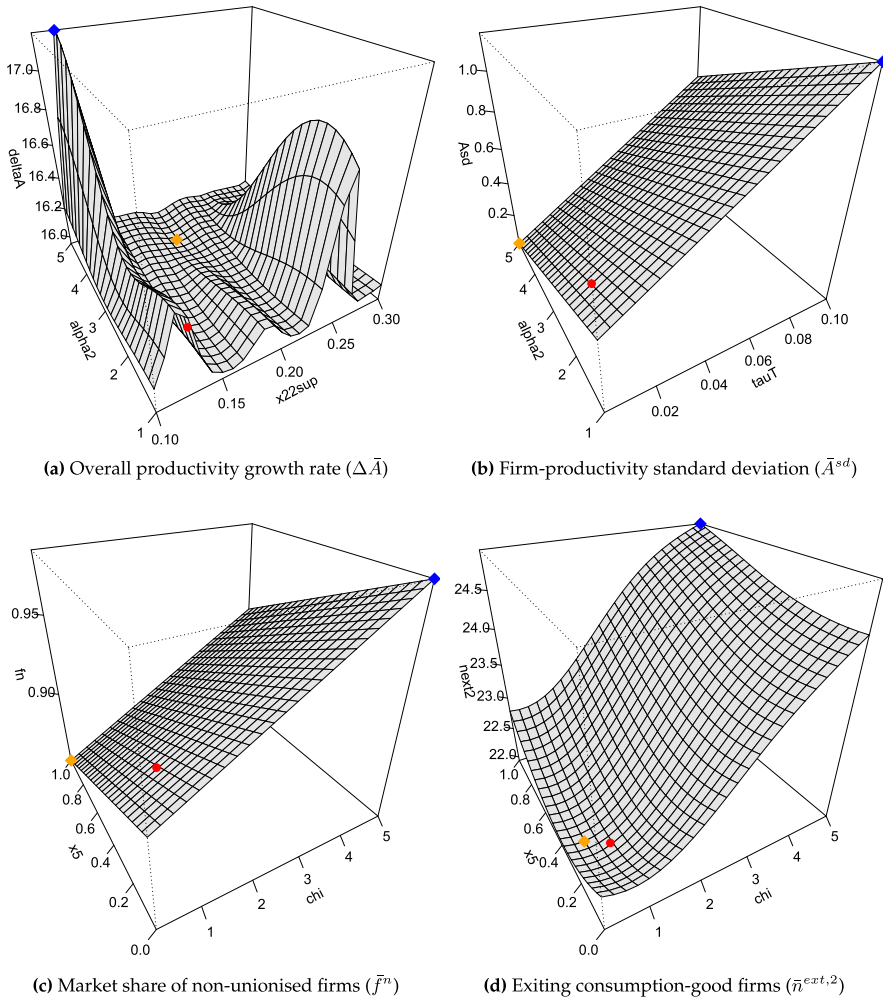


Figure C4 Global sensitivity analysis response surfaces. Modeled using fitted Kriging meta-model. Red dot: calibration settings—Markers: maximum (blue) and minimum (orange) predicted values.

Overall, the impacts of all the tested factors in the SVD are relatively mild, and the differences between the two scenarios are small.²¹ Figure C4 presents an exploration of the Kriging meta-model response surface for the two most relevant factors for each metric. The flat surfaces in Figure C4(b) and C.4(c) indicates the linear interaction of the response with the factors. Productivity growth ($\Delta \bar{A}$) shows the most rugged surface, indicating non-linear interactions in the two factors, despite the small amplitude from the minimum to maximum (below 7%). Additionally, just two factors, χ and x_5 , account for more than 40% of the estimated meta-model effects on the market-dynamics metrics $\bar{f}^n$ and $\bar{n}^{ext,2}$. However, also for these two variables, the maximal variation of the effects is mild (both below 14%). Therefore, for those three metrics, the model results are qualitatively robust to *any* parametric configuration.

21 The response surfaces selected for Figure C4 are the ones most affected in both scenarios by the indicated factors, which, in turn, are the two most relevant factors for each metric. Figures C4(a), C4(b), and C4(d) are from the no-invasion case, while C4(c) is from the invasion one.

Table C2. Stock-and-flow consistency: transaction flow matrix. (*) Government deficit/superavit is close to zero in the long run

	Workers	Capital-good firms		Consumption-good firms		Bank		Government	Σ
		Current	Capital	Current	Capital	Current	Capital		
Consumption	$-C$	$+C$							0
Investment		$+I$			$-I$				0
Govt. expenditures	$+G$							$-G$	0
Wages	$+W$	$-W^1$		$-W^2$					0
Profits, firms		$-\Pi^1$	$+\Pi^1$	$-\Pi^2$	$+\Pi^2$				0
Profits, bank						$-\Pi^b$	$+\Pi^b$		0
Debt interests		$-r\text{Deb}_{t-1}^1$		$-r\text{Deb}_{t-1}^2$		$+r\text{Deb}_{t-1}$			0
Deposits interests		$+r\text{NW}_{t-1}^1$		$+r\text{NW}_{t-1}^2$		$-r\text{NW}_{t-1}$			0
Taxes		$-\text{Tax}^1$		$-\text{Tax}^2$				$+\text{Tax}$	0
Change in debt			$+\Delta\text{Deb}^1$		$+\Delta\text{Deb}^2$		$-\Delta\text{Deb}$		0
Change in deposits			$-\Delta\text{NW}^1$		$-\Delta\text{NW}^2$		$+\Delta\text{NW}$		0
Σ	0	0	0	0	0	0	0	0*	0*

The only metric for which factors influence is *qualitatively relevant* is the firm-productivity standard deviation (A^{sd}), as shown by Figure C4(b). Firstly, it shows the strong impact of worker-skill accumulation, driven by parameter τ_T , on the dispersion of the firm-level productivities. This hints at the significant consequences, in terms of firm heterogeneity, of the interaction of learning and labor market processes in the model. Secondly, the importance of α_2 , or the minimum technological capabilities of entrant firms, in the classical “creative destruction” Schumpeterian sense. Indeed, this dispersion metric is very sensitive to changes of the two factors, additively, indicating that the selected calibration point ($\tau_T = 0.01$, $\alpha_2 = 2$) is quite conservative in terms of the possibilities of the K + S model in reproducing the empirical stylized fact of the sustained heterogeneity among firms.